

HILLCREST HIGH SCHOOL



GRADE 12

MATHEMATICS

PAPER 2

JUNE 2014

MARKS: 125

TIME: 2½ Hours

This question paper consists of 9 pages and a formula sheet

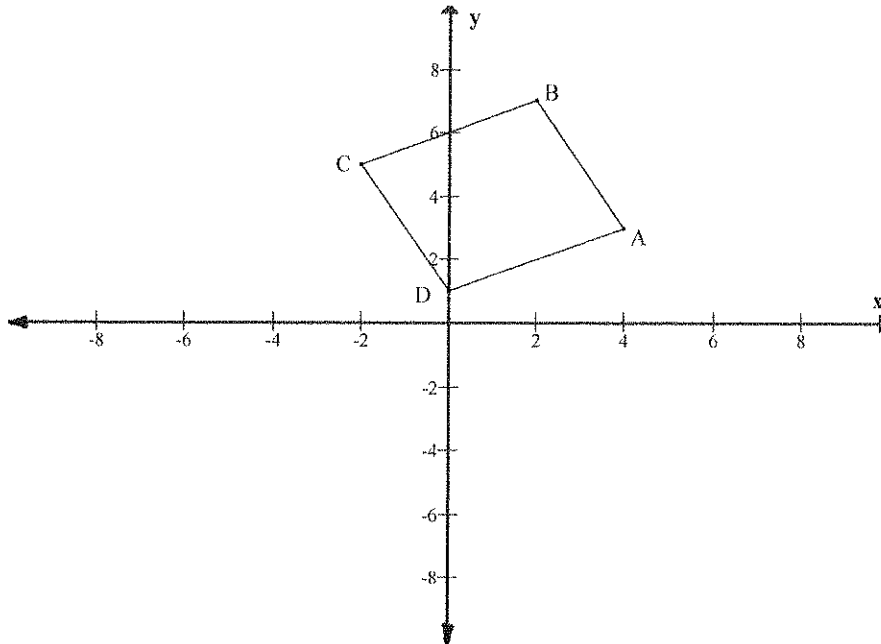
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 8 questions. Answer ALL the questions.
2. Show ALL calculations, diagrams, graphs etc., clearly, which you have used in determining the answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Number the answers correctly according to the numbering system used in this question paper.
6. Diagrams are NOT necessarily drawn to scale.
7. Write neatly and legibly.

Question 1

1. In the diagram A(4;3) B(2;7) C(-2;5) and D(0;1) are four points on a Cartesian plane.



- 1.1 Calculate the length of BD (leave your answer in surd form if necessary). (3)
- 1.2 Show that $CA = BD$ (2)
- 1.3 Show that the co-ordinates of M, the midpoint of BD are (1;4) (3)
- 1.4 Prove that $AM \perp BD$ (5)
- 1.5 Prove that A, M, and C are collinear. (3)
- 1.6 State, giving reason, which type of quadrilateral ABCD is. (2)

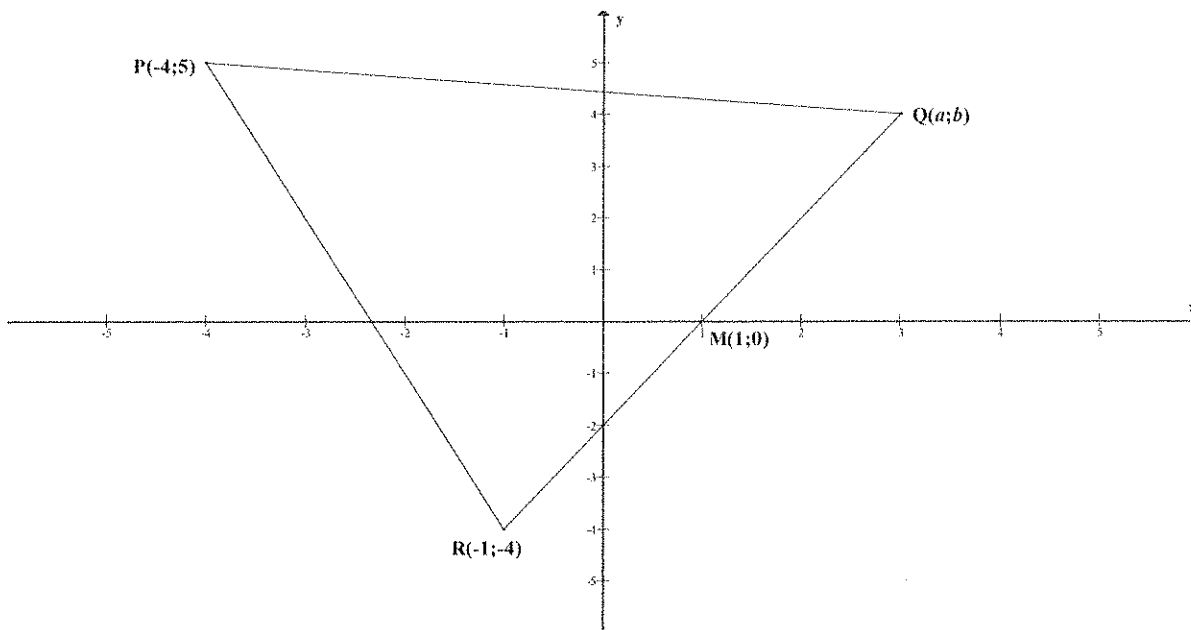
[18]

Question 2

- 2.1 Determine the equation of the straight line on a Cartesian plane, intercepting the X-axis at 3 and the Y-axis at -4. (3)
- 2.2 The straight line $ax + by + 2 = 0$ makes an angle of 135° with the positive X-axis and passes through the point (2;-1).
Calculate the values of a and b . (5)

2.3 In the given diagram, $P(-4;5)$, $Q(a;b)$ and $R(-1;-4)$ are the vertices of ΔPQR .

$M(1;0)$ is the midpoint of QR .



2.3.1 Determine the values of a and b . (2)

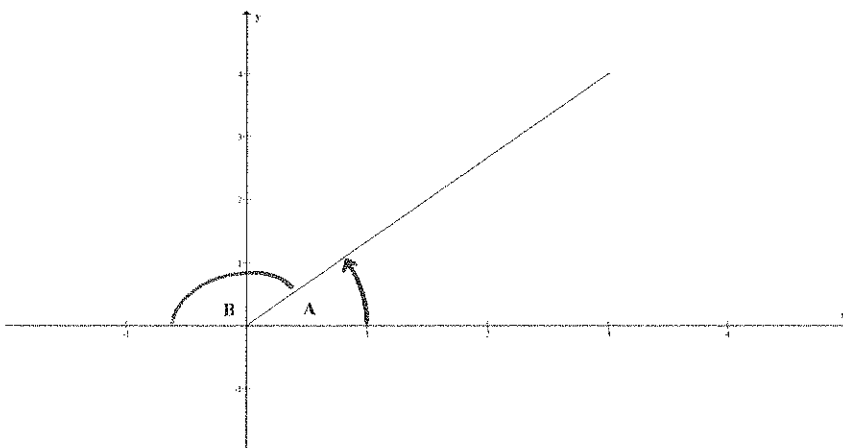
2.3.2 Determine the equation of the perpendicular bisector of QR in the form

$$y = mx + c. \quad (5)$$

[15]

Question 3

3.1 If $\hat{A} + \hat{B} = 180^\circ$ and $5 \cos A - 3 = 0$, without the use of a calculator, determine:



3.1.1 $\sin A$ (2)

3.1.2 $\cos B$ (2)

3.1.3 $\cos 2A$ (3)

3.2 Use a sketch to prove that for all values of θ : $\sin^2\theta + \cos^2\theta = 1$ (3)

3.3 Simplify as far as possible:

$$\frac{\sin(180 + A) \sin(90 - A)}{\cos(180 - A) \cos(-A) \tan(360 - A)}$$

(5)

3.4.1 Prove that:

$$\sin 3A = 3 \sin A - 4 \sin^3 A$$

(5)

3.4.2 Hence, determine the general solution for A if:

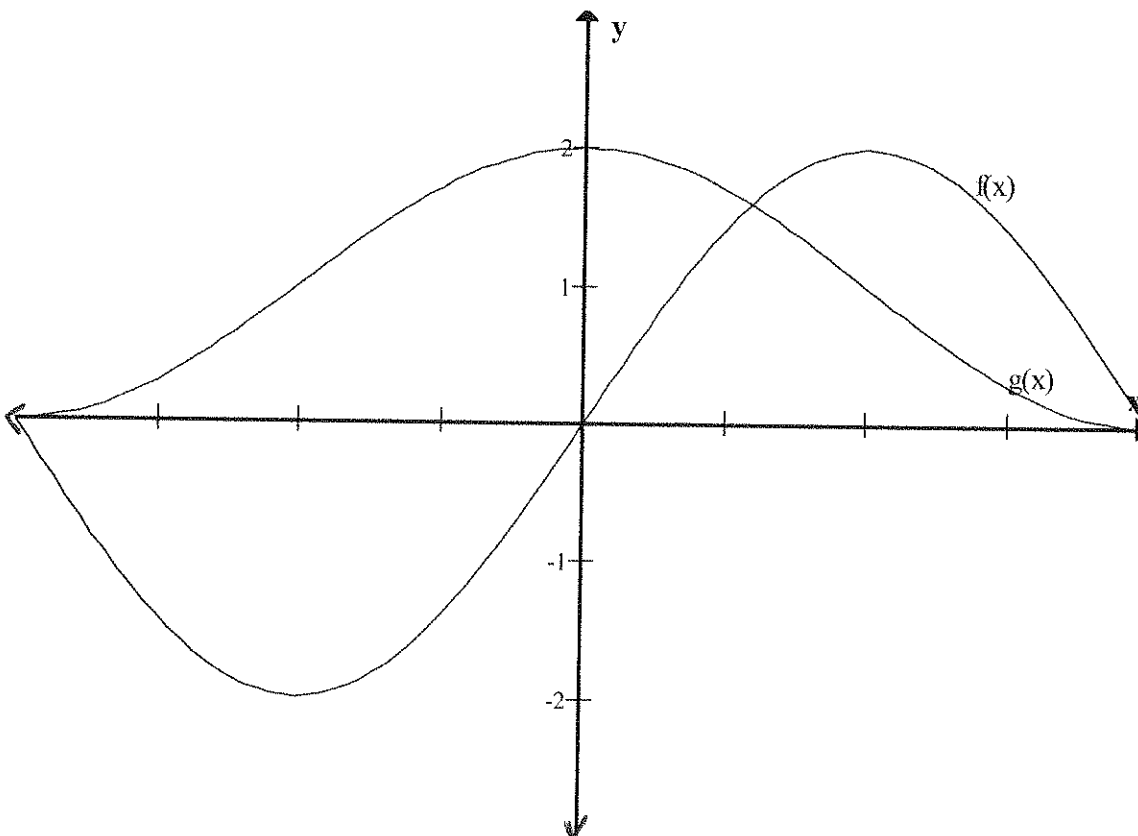
$$8 \sin^3 A = 6 \sin A + 1$$

(7)

$$-1 = 6 \sin A - 8 \sin^3 A$$

[27]

Question 4



$f(x) = 2 \sin x, x \in [-180^\circ; 180]$ and $g(x) = \cos x + 1, x \in [-180^\circ; 180]$

4.1 What is the period of f ? (1)

4.2 What is the amplitude of g ? (1)

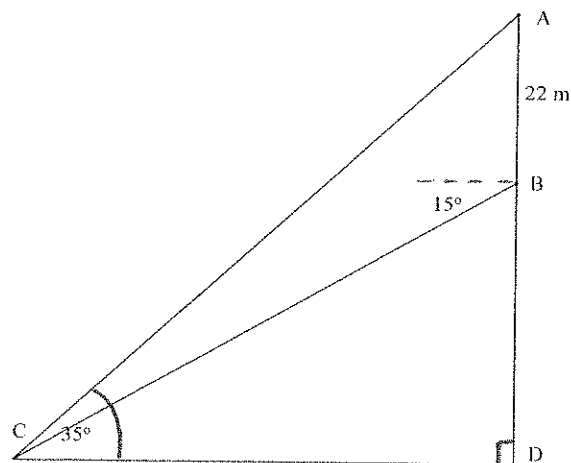
4.3 Use your graph and determine the value(s) of x for which:

$$\sin 2x + 2 \sin x = 0 \quad (5)$$

[7]

Question 5

5.1 A man, B, stands on the observation deck of a lighthouse. The angle of depression of a child, C, is 15° . The foot of the lighthouse, D, and the child C, are on the same horizontal plane. The top of the lightening conductor, A, is at an angle of elevation of 35° from the child, C. The distance from the man, B, to the top of the lightening conductor A, is 22m.

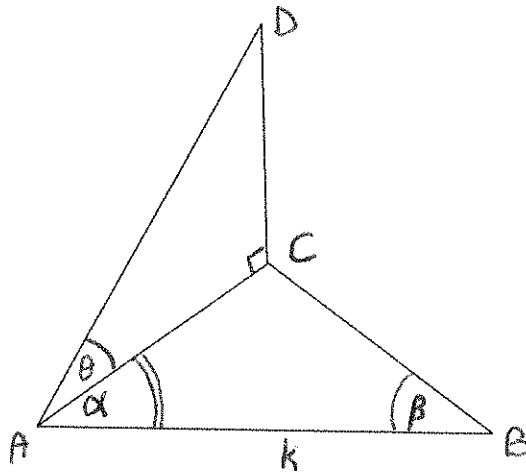


Calculate, rounded off to one decimal place:

5.1.1 The distance from the child, C to the man, B (i.e. BC) (5)

5.1.2 How far is the child from the foot of the lighthouse? (i.e. CD) (3)

- 5.2 CD is a vertical pole. A wire is stretched from D to A. A, B and C are points on the same horizontal plane. $AB = k$; $\hat{CAB} = \alpha$; and $\hat{ABC} = \beta$. The angle of elevation of D from A is ϑ .



Prove that:

$$DA = \frac{k \sin \beta}{\sin(\alpha + \beta) \cdot \cos \vartheta}$$

(5)

[13]

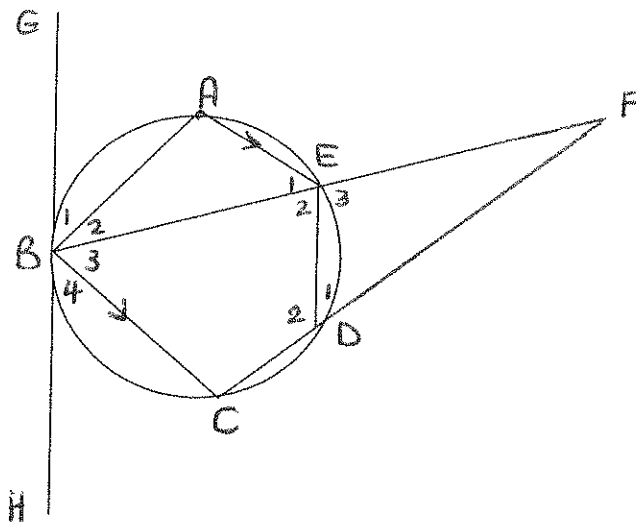
Question 6

6.1 Complete the following statement:

“The angle between the tangent and the chord at the point of contact is equal to _____”

(1)

- 6.2 In the diagram, A, B, C, D and E are points on the circumference of the circle such that $AE \parallel BC$. BE and CD produced meet in F. GBH is a tangent to the circle at B. $\hat{B}_1 = 68^\circ$ and $\hat{F} = 20^\circ$



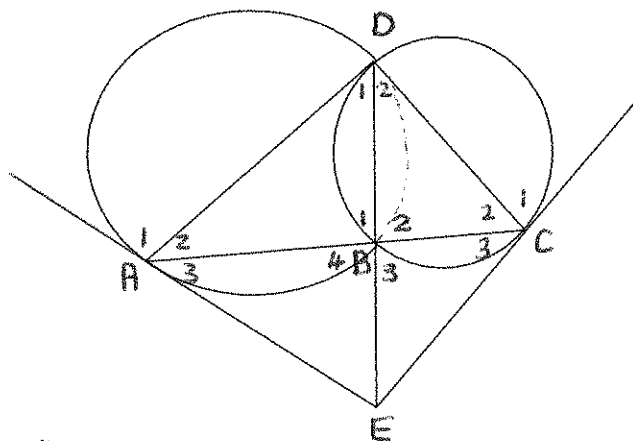
Determine with reasons:

- 6.2.1 \hat{E}_1 (2)
- 6.2.2 \hat{B}_3 (2)
- 6.2.3 \hat{D}_1 (2)
- 6.2.4 \hat{E}_2 (2)
- 6.2.5 \hat{C} (2)

[11]

Question 7

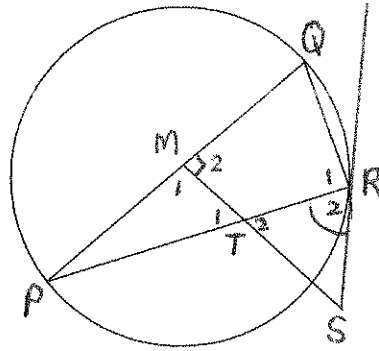
- 7.1 Prove the theorem that states that the opposite angles of a cyclic quadrilateral are supplementary. (7)
- 7.2 Now write down two other ways of proving that a quadrilateral is cyclic. (2)
- 7.3 In the diagram below, two circles intersect at D and B. ABC is a straight line and the tangents to the circles at A and C meet DB produced at E. (2)



Prove:

- 7.3.1 ADCE is a cyclic quadrilateral (5)
- 7.3.2 EA = EC (4)

7.4

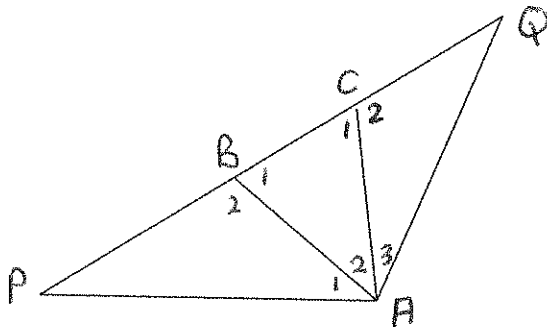


In the diagram, M is the centre of the circle, $SM \perp$ to the diameter PQ. RS is a tangent to circle at R. PR and SM intersect at T. Prove that ΔSTR is an isosceles triangle. (6)

[24]

Question 8

$\hat{P}AQ = 120^\circ$, ΔABC is equilateral. PBCQ is a straight line.



Prove that:

8.1 $\Delta PBA \parallel \Delta ACQ$ (6)

8.2 $PB \cdot CQ = BC^2$ (4)

[10]