

HILLCREST HIGH SCHOOL



Grade 12 Mathematics Paper 1

MARKS: 125

TIME: 2 and 1/2 hours

INSTRUCTIONS

1. This question paper consists of TEN questions. Answer ALL the questions.
2. Number the questions correctly according to the number system used in this question paper.
3. Start EACH question on a NEW page.
4. You may use an approved calculator (non-programmable and non-graphical), unless stated otherwise.
5. Show ALL calculations clearly. Answers only will not be awarded full marks.
6. Round off ALL final answers to TWO decimal places, unless stated otherwise.
7. Indicate units of measurement, where applicable.
8. Maps and diagrams are NOT necessarily drawn to scale, unless otherwise stated.
9. Write neatly and legibly.

QUESTION 1:

1.1 Solve for x :

1.1.1 $3x^2 - 2x = 0$ (2)

1.1.2 $x - \frac{2}{x} = 5; x \neq 0$. (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $x^{\frac{3}{2}} = 8$ (2)

1.2 Simplify, without the use of a calculator:

$\sqrt{3} \cdot \sqrt{48} - \frac{4^{x+1}}{2^{2x}}$ (3)

1.3 The solution to a quadratic equation is $x = \frac{3 \pm \sqrt{4-8p}}{4}$ where $p \in \mathbb{Q}$.

Determine the value(s) of p such that:

1.3.1 The roots of the equation are equal (2)

1.3.2 The roots of the equation are non-real. (2)

[15]

QUESTION 2:

2.1 Given: $f(x) = x^2 + x$, determine $f'(x)$ from first principles. (5)

2.2 Determine:

2.2.1 $\frac{dy}{dx}$ if $y = x^3 - x - \sqrt{x}$ (4)

2.2.2 $f'(x)$ if $f(x) = \frac{8x^3-1}{2x-1}$ (4)

[13]

QUESTION 3:

Given the equation of the curve $h(x) = (2x - 1)(x - 3)^2$.

3.1 If $(-1; b)$ lies on the curve of h , calculate the value of b . (2)

3.2 Hence, determine the equation of the tangent to $h(x)$ at $x = -1$. (5)

[7]

QUESTION 4:

Given: $g(x) = -2x^3 - 3x^2 + 12x + 20$

4.1 Calculate the intercepts of the graphs of g with the axes. (5)

4.2 Determine the co-ordinates of the turning point of g . (5)

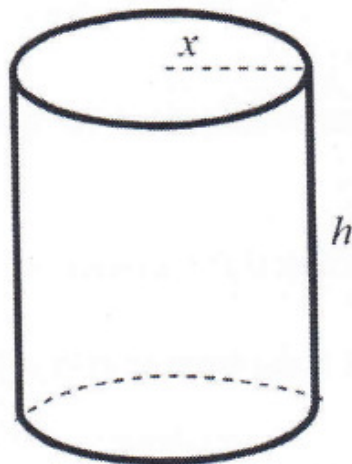
4.3 Sketch the graph of g , clearly indicating the intercepts with the axes and its turning points. (5)

4.4 Determine the value(s) of x for which $g(x)$ is decreasing. (2)

[17]

QUESTION 5:

A cylindrical prism with no top has a volume of 340 ml, radius (r) equal to x cm and height h cm. $1\text{ml} = 1\text{cm}^3$



5.1 Write down the height in terms of x . (2)

5.2 Show that the surface area(S) of the cylindrical prism with no top is given by

$$S = \pi x^2 + \frac{680}{x} \quad (2)$$

5.3 Determine the value of x for which the least amount of metal is required to manufacture the cylindrical prism. (4)

[8]

QUESTION 6:

- 6.1 Given the arithmetic series: $- 6 + 0 + 6 + \dots + 288$
- 6.1.1 Determine the number of terms in this series. (3)
- 6.1.2 Calculate the sum of this series. (3)
- 6.1.3 Calculate the sum of all the whole numbers up to and including 288 that are divisible by 9. (6)
- 6.2 The first three terms of an infinite geometric sequence are 6, 3 and $1\frac{1}{2}$ respectively.
- 6.2.1 Determine the general term of the sequence. (2)
- 6.2.2 Which term has a value of $\frac{3}{16}$? (4)
- 6.2.3 Calculate the sum to infinity of this sequence. (2)
- [20]**

QUESTION 7:

- 7.1 A given quadratic pattern $T_n = an^2 + bn + c$ has $T_2 = T_4 = 0$ and a second difference of 12. Determine the value of the 3rd term of the pattern. (7)

- 7.2 Evaluate:

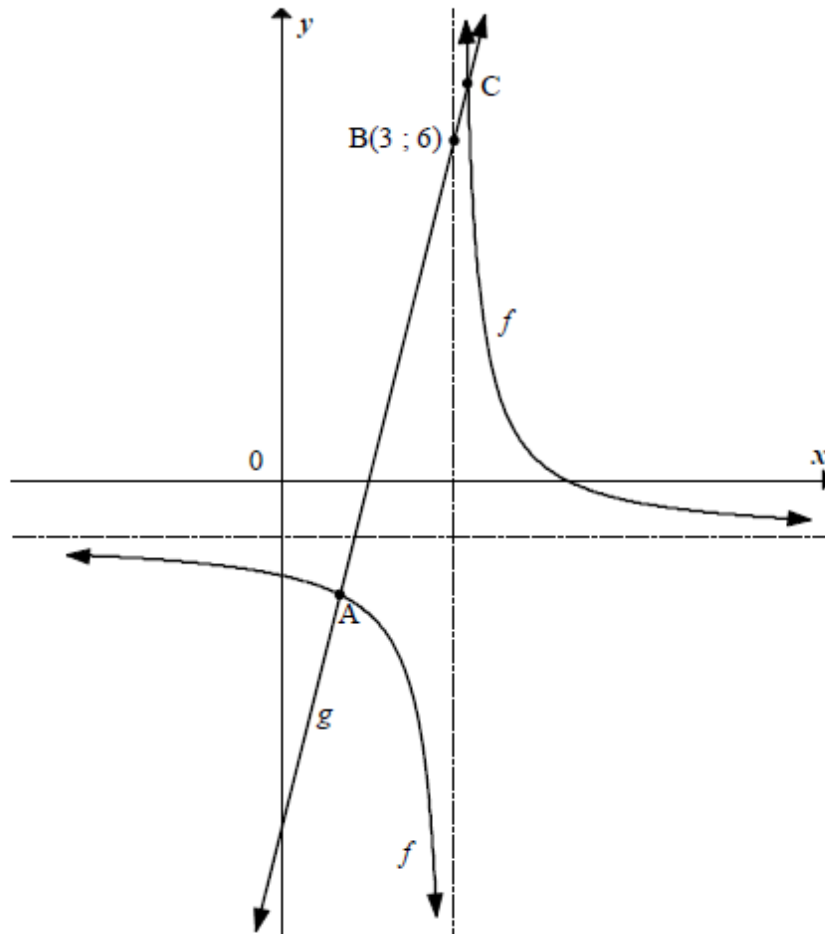
$$\sum_{k=3}^5 (-1)^k \frac{2}{k}$$

(4)
[11]

QUESTION 8:

The sketch below represents the graphs of $f(x) = \frac{2}{x-3} - 1$ and $g(x) = dx + e$.

Point B (3 ; 6) lies on the graph of g and the two graphs intersect at points A and C.



- 8.1 Write down the equations of the asymptotes of f . (2)
- 8.2 Write down the domain of f . (2)
- 8.3 Determine the values of d and e , correct to the nearest integer, if the graph of g makes an angle of 76° with the positive x -axis. (3)
- 8.4 Determine the coordinates of A and C. (6)
- 8.5 For what values of x is $g(x) \geq f(x)$? (3)
- 8.6 Determine an equation for the axis of symmetry of f which has a positive slope. (3)
- [19]

QUESTION 9:

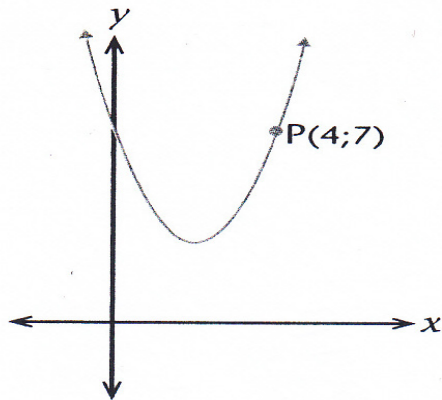
Sketch the graph of $f(x) = ax^2 + bx + c$ if it is also given that:

- The range of f is $(-\infty; 7]$
- $a \neq 0$
- $b < 0$
- One root of f is positive and the other root of f is negative.

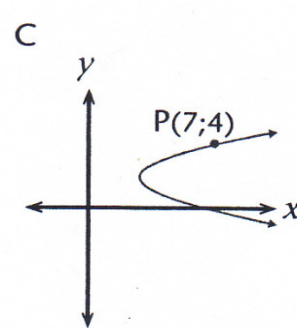
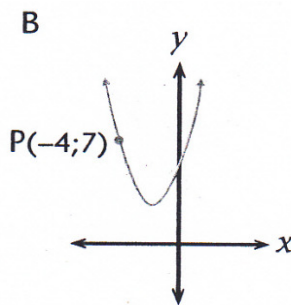
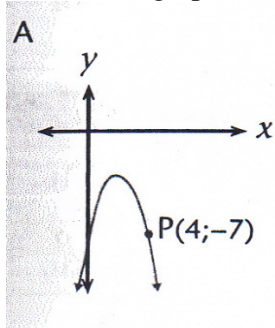
[4]

QUESTION 10:

10.1 The figure below shows the graph of



The graphs below represent _____ and _____



10.1.1 Match the graph with its function and write down the letter corresponding to each function. (3)

10.1.2 State which graph (A, B or C) is not a function and how the range should be restricted so that it becomes a function. (2)

10.2 Consider the functions _____ and _____ .

10.2.1 Determine _____ in the form _____ (2)

10.2.2 State the x – intercept of _____ (2)

10.2.3 Determine _____ if _____ is the reflection of f about the x -axis. (2)

[11]