HILLCREST HIGH SCHOOL

Grade 10 Exam
June 2017

Examiner: Mrs Sparks
MARKS: 100

Moderator: Mrs Woodrow
TIME: 2 hours

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 7 questions. Answer ALL the questions.
2. Write your name and **your Mathematics teacher**'s name on your answer booklet.
3. Clearly show ALL calculations, diagrams, graphs, etc which you have used to determine your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable) may be used, unless otherwise stated.
6. If necessary, answers should be rounded off to TWO decimal places, unless otherwise stated.
7. Number the answers EXACTLY as the questions are numbered.
8. Diagrams are not necessarily drawn to scale.
9. It is in your own interest to write legibly and to present your work neatly.

QUESTION 1

1.1 Simplify

1.1.1 $(3m + 2n)(9m^2-6mn + 4n^2)$ (2)

1.1.2 $3^{x-1}.3^{-x+1}$ (2)

1.2 If $x^{-2} = 9$ and $y^{-2} = 16$, where $x, y > 0$, calculate $(x^{-1} + y^{-1})^2$ (4)

[8]

QUESTION 2

2.1 Solve for the unknown variable:

2.1.1 $x(x + 6) = 16$ (3)

2.1.2 $-3(2y + 5) > y + 6$ and $y \in [-8; \infty)$

(a) List the solution if $y \in Z$ (3)

(b) Represent the solution graphically (on a number line) if $y \in R$ (2)

2.2 Given: $\frac{2n-7}{6}+2=\frac{7n}{3}-1$

2.2.1 Solve n to 5 decimal places. (3)

2.2.2 Is n a rational or irrational number? Motivate. (2)

2.3 Use the following equations to solve for x and y simultaneously:

$$2x - 3y = 14; \quad x - 5y = 0$$ (4)

2.4 Ship A leaves a harbour and sails at 20 km/h. Ship B leaves the same harbour 20 minutes later and sails at 24 km/h in the same direction. If all weather and conditions at sea remain constant determine the following:

2.4.1 The distance ship A covers, in terms of x, if you let the time it takes for the ships to meet be x? (1)

2.4.2 How many kilometres would ship B travel in 20 minutes? (1)

2.4.3 Determine x (the length of time taken for the ships to meet). (2)

[21]

QUESTION 3

3.1 Given the sequence: -10; -6; -2; …

Determine

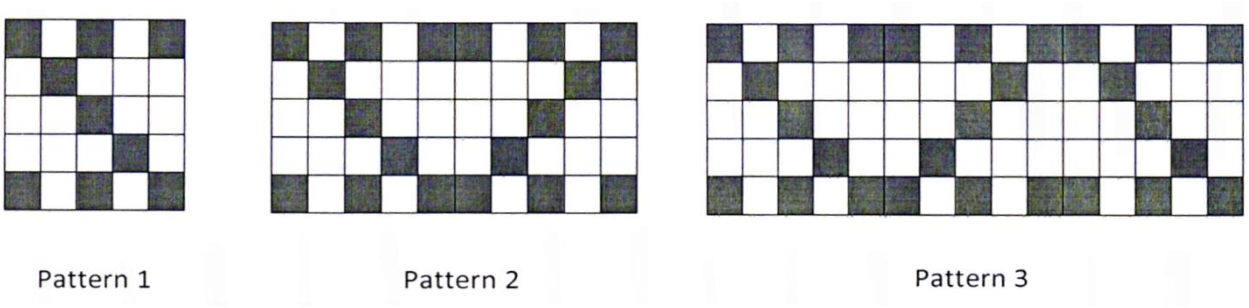
3.1.1 The next two terms in the sequence. (1)

3.1.2 The n^{th} term of the sequence. (2)

3.1.3 The sum of the first 6 terms of the sequence. (2)

3.1.4 Which term is equal to 78. (2)

3.2 Consider the geometric pattern given below:



Pattern 1 Pattern 2 Pattern 3

3.2.1 How many black tiles will there be in the 5^{th} pattern? (1)

3.2.2 How many more white tiles than black tiles will there be in the 9^{th} pattern? (2)

[10]

QUESTION 4

4.1 Given the functions $f(x) = \frac{3}{x} + 1$ and $g(x) = -\frac{1}{9}x^2 + 1$

4.1.1 Write down the equations of the asymptote(s) of f. (2)

4.1.2 Determine the x-intercepts of g. (3)

4.1.3 Draw neat sketch graphs of f and g over the set of real numbers on DIAGRAM SHEET 1. Clearly indicate all critical points and any asymptotes on your graph. (6)

4.1.4 Is f an increasing or decreasing function? (1)

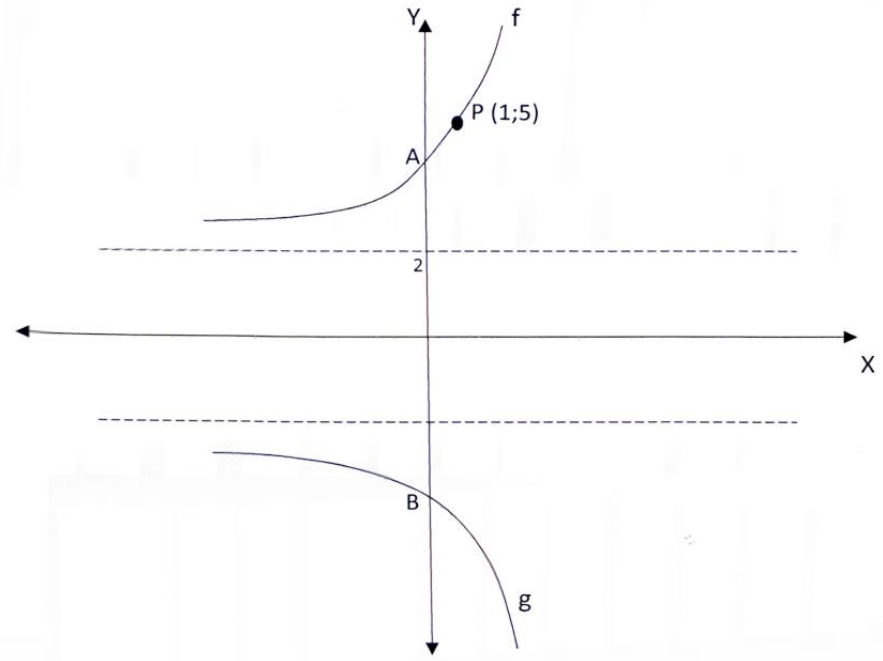
4.1.5 Determine the range of g. (1)

4.1.6 Determine the domain of f. (1)

4.1.7 (-3;0) and (0;1) are two points on the straight line m. Determine the equation of m in the form y = … (3)

4.1.8 P(x; y) is a common point of f and m. Determine the co-ordinates of P if $x > 0$. (3)

4.2 The diagram given below represents the graphs of the functions $f(x) = b^x + q$ and g, the reflection of f about the x-axis. P (1;5) is a point on f.



4.2.1 Determine the values of b and q. (3)

4.2.2 Determine the equation that defines g. (2)

4.2.3 Calculate the co-ordinates of A and hence the length of AB. (3)

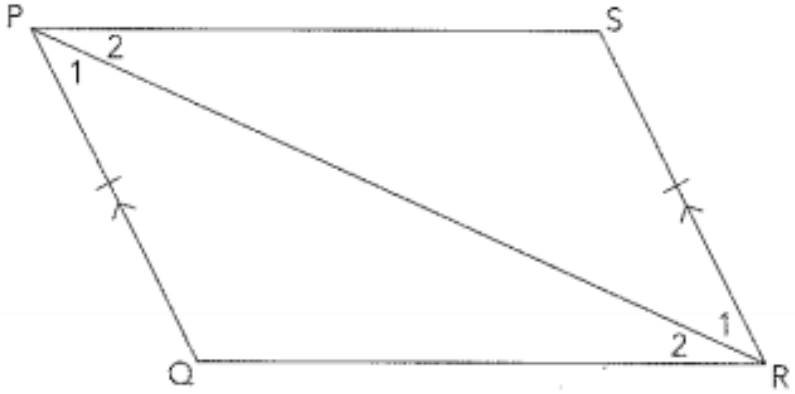
4.2.4 If f is reflected about the y-axis and then translated three units down, give the equation of the new function, h, in the form y = … (2)

[30]

ANSWER QUESTIONS 5 to 7 ON THE DIGRAM SHEETS ATTACHED TO YOUR ANSWER BOOKLET

QUESTION 5

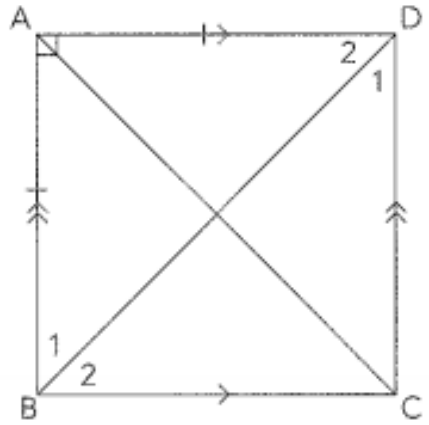
In the diagram, PQ = RS and PQ∥RS.



Use the diagram above to prove the theorem that states: if a quadrilateral has one pair of opposite sides equal and parallel then the quadrilateral is a parallelogram. **[5]**

QUESTION 6

6.1 In the diagram, AB = AD, AB∥CD, AD∥BC and $B\hat{A}D = 90°$.

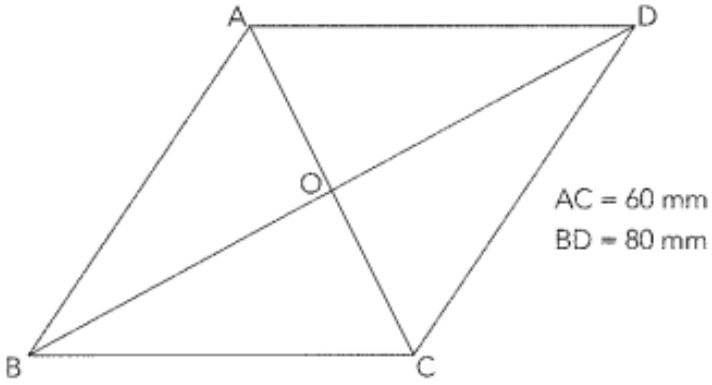


Give reasons for the following statements in your answer booklet.

	Statement
6.1.1	$\hat{B}_1 + \hat{D}_2 + B\hat{A}D = 180°$
6.1.2	$\hat{B}_1 = \hat{D}_2$
6.1.3	$\hat{D}_1 = \hat{B}_1$
6.1.4	ABCD is a paralleogram
6.1.5	ABCD is a square

(5)

6.2 In the sketch below ABCD is a rhombus with AC = 60mm and BD = 80mm.



6.2.1 Give the size of $A\hat{O}D$, with reasons. (2)

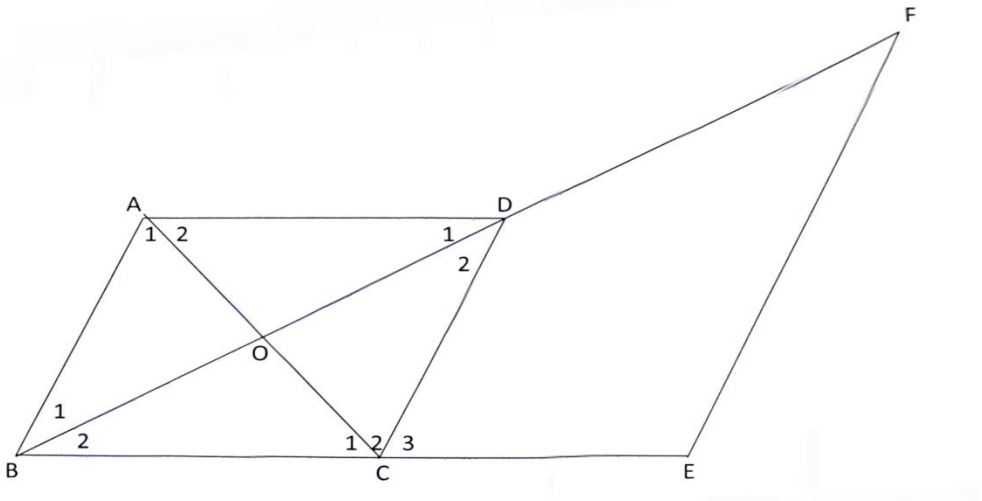
6.2.2 Calculate the length of AD, with reasons. (4)

[11]

QUESTION 7

7.1 Complete the statement of the Midpoint Theorem: The line segment joining the midpoints of two sides of a triangle _____ (2)

7.2 In the diagram below: ABCD is a rhombus with diagonals intersecting in O. BD is produced to F and BC is produced to E, such that BE = EF. EF is drawn. C and D are the midpoints of BE and BF.



Prove, with reasons:

7.2.1 AB //EF (2)

7.2.2 2AD = EF (2)

7.2.3 $\hat{D}_1 = \hat{F}$ (2)

Is the following statement true or false? Motivate.

7.2.4 △ABD ||| △EBF (4)

7.2.5 If $\hat{F} = 35°$ calculate, with reasons, the size of $\hat{C}_1$. (3)

[15]

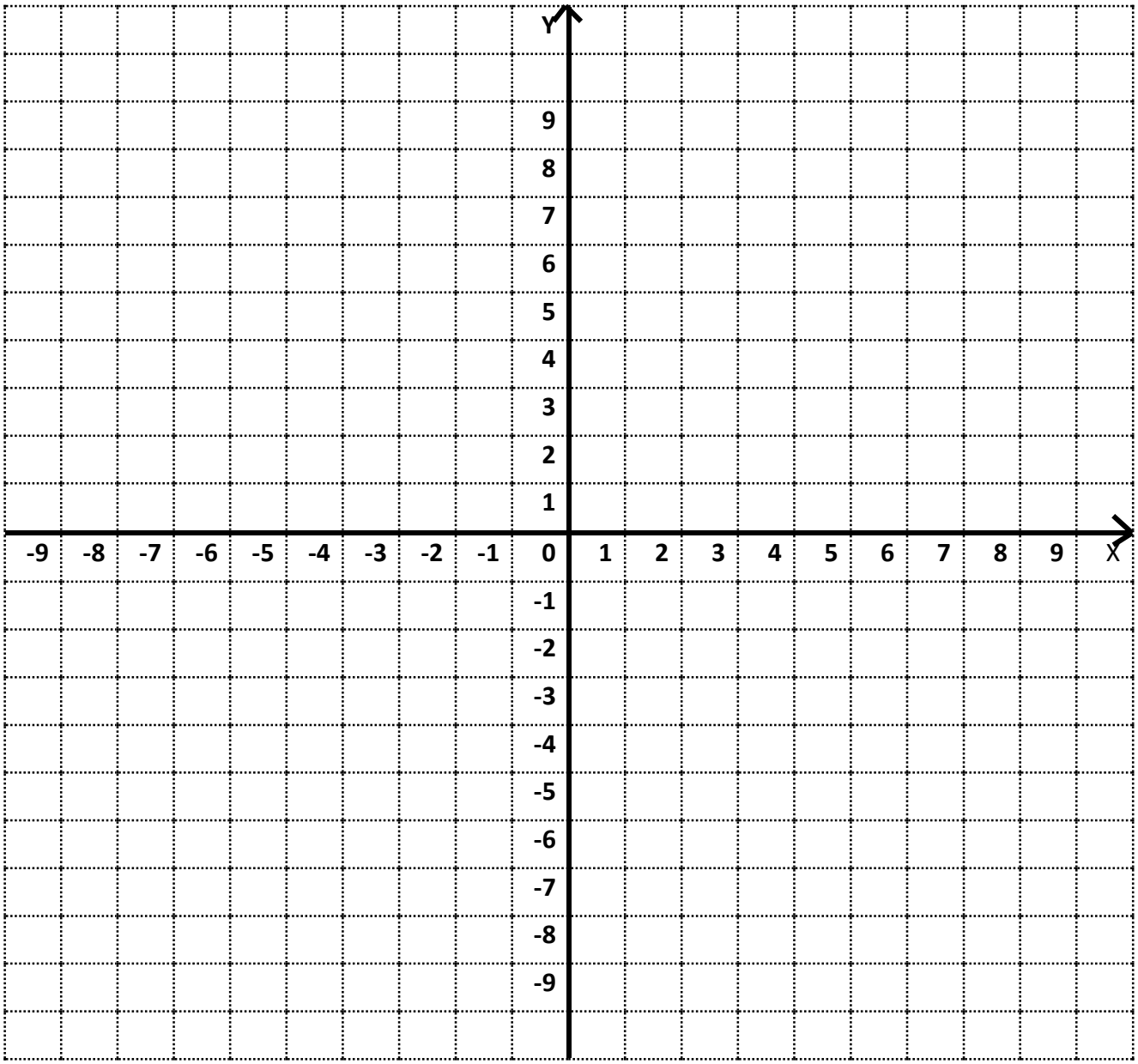
THE END

GRADE 10 MATHEMATICS JUNE 2017

Name: _____ Teacher: _____

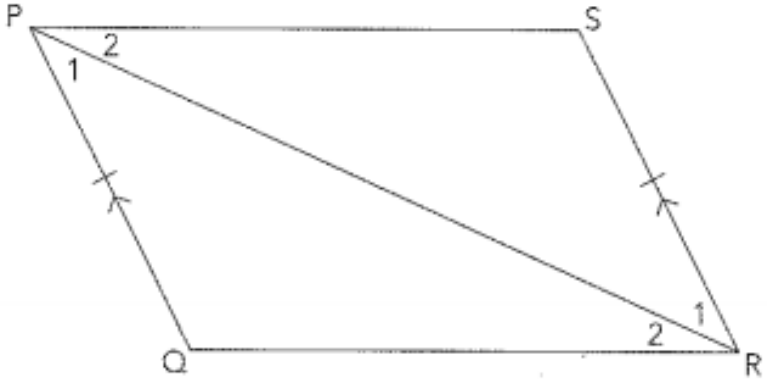
Question #	1	2	3	4	5	6	7	Total
Mark								
Out of	8	21	10	30	5	11	15	100
Signed								

QUESTION 4.1.3



ANSWER BOOKLET FOR GEOMETRY QUESTIONS 5 to 7

QUESTION 5



RTP: PQRS is a parm

PROOF: _____

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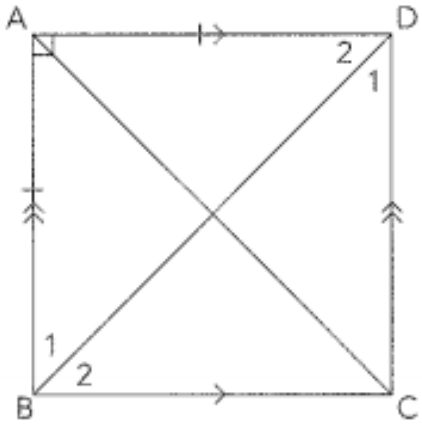
_____

_____(5)

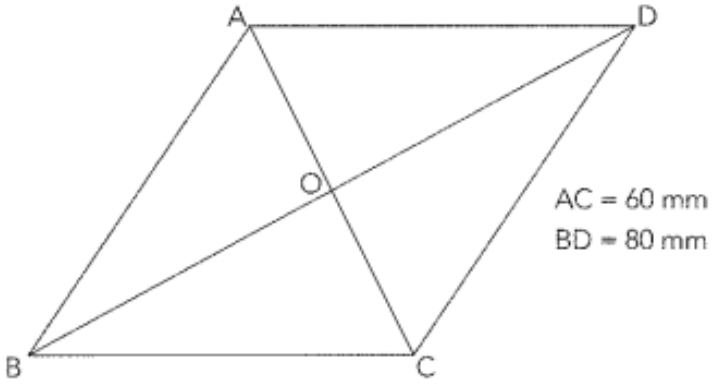
QUESTION 6



	Statement	Reason
6.1.1	$\hat{B}_1 + \hat{D}_2 + B\hat{A}D = 180°$	
6.1.2	$\hat{B}_1 = \hat{D}_2$	
6.1.3	$\hat{D}_1 = \hat{B}_1$	
6.1.4	ABCD is a paralleogram	
6.1.5	ABCD is a square	

(5)

6.2

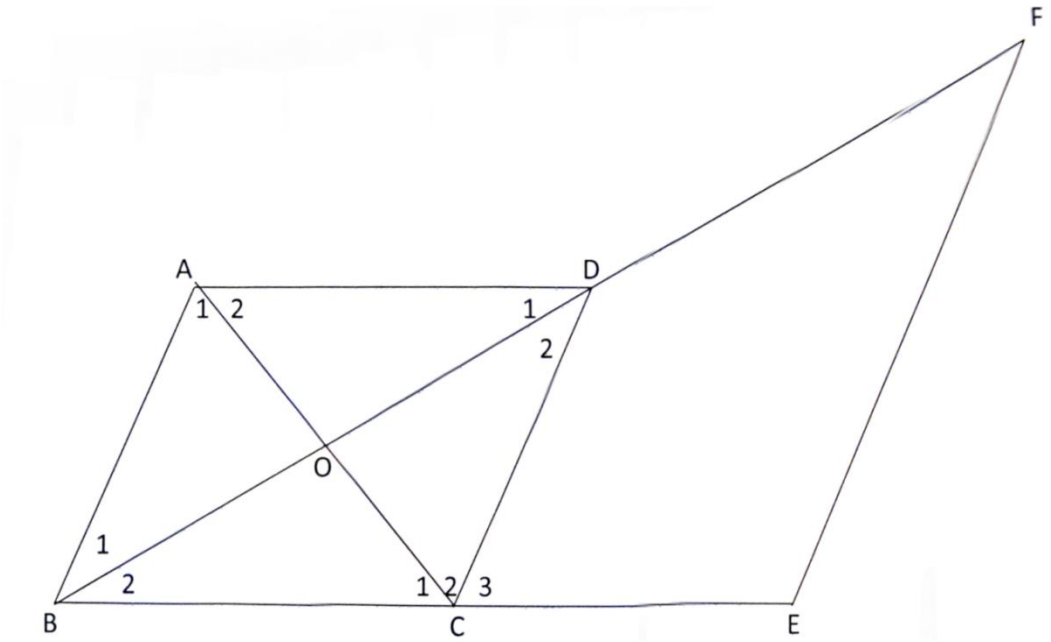


6.2.1		
		(2)

6.2.2		
		(4)

QUESTION 7

7.1	The line segment joining the midpoints of two sides of a triangle	
		(2)



7.2.1		(2)

7.2.2		(2)

7.2.3		(2)

7.2.4		(4)

7.2.5		(3)

[15]

MATHEMATICS

GRADE 10 EXAMINATION

JUNE 2017

MEMORANDUM

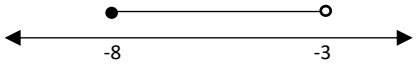
QUESTION 1

1.1.1	$(3m + 2n)(9m^2 – 6mn + 4n^2)$ $= 27m^3+8n^3$	√ **$27m^3$** √ $8n^3$ (2)
1.1.2	$3^{x-1}.3^{-x+1}$ $= 3^0$ =1	√ adding exponents √ **1** (2)
1.2	Given: $x^{-2} = 9$ and $y^{-2} = 16$ $(x, y > 0)$ $\therefore\ x^{-1} = 3$ and $y^{-1} = 4$ $\therefore\ (x^{-1} + y^{-1})^2 = x^{-2} + 2x^{-1}y^{-1} + y^{-2}$ $= 9 + 2(3)(4) + 16$ $= 49$ OR	**m√ prime factors** √ $x^{-1} = 3$ and $y^{-1} = 4$ Or x = 1/3 and y = 1/4 √ substitution √ answer (4)

[8]

QUESTION 2

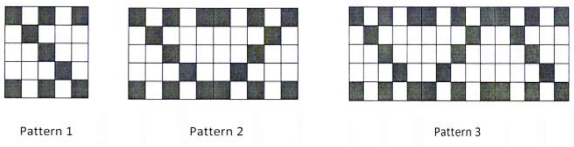
2.1.1	$x(x + 6) = 16$ $x^2 + 6x – 16 = 0$ $(x + 8)(x – 2) = 0$ $x = -8$ or $x = 2$	√ standard form √ factors **ca**√ answer (3)
2.1.2 (a)	$-3(2y + 5) > y + 6$ and $y \in [-8;\infty)$; $y \in Z$ $-6y - 15 > y + 6$ $-7y > 21$ $y < -3$ Solution: -4;-5;-6;-7;-8	√ $-6y - 15 > y + 6$ √ $y < -3$

		ca√ -4;-5;-6;-7;-8 (3)
2.1.2 (b)	$y \in R$  	√ -8 and solid dot √ -3 and hollow dot (2)
2.2.1	$\frac{2n-7}{6}+2=\frac{7n}{3}-1$ (LCD: X 6) ∴ 2n – 7 +12 = 14n – 6 ∴ -12n = -11 $n=\frac{11}{12}$ = 0,91667	√ 2n – 7 +12 = 14n – 6 √ n=11/12 ca√ answer (5 dec) (3)
2.2.2	Rational number $\frac{11}{12}=0,91666\ldots$ is a recurring decimal fraction/ can be expressed as a fraction	√ Rational number √ reason (2)
2.3	$2x-3y=14 \ldots\ldots(1)$ $x-5y=0 \ldots\ldots(2)$ $x=5y \ldots..(2)$ $2(5y)-3y=14$ $7y=14$ $y=2$ When y=2 $x=5(2)$= 10	√ M isolating one variable √ substitution √a y = 2 √ca x = 10 (4)
2.4.1	Distance Ship A = 20x	√a (1)
2.4.2	20min = 8km	√a (1)
2.4.3	Distance Ship B = 24x - 8 $20x=24x-8$ $8=4x$ $x=2$ $\therefore 2\ hours$	√ Equation √ 2 hours **(no unit, no mark)** (2)

[21]

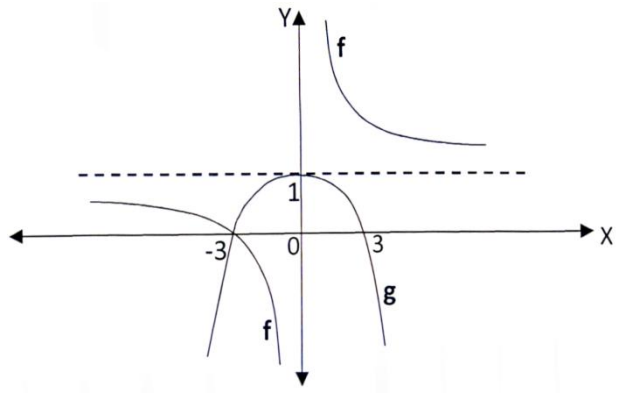
QUESTION 3

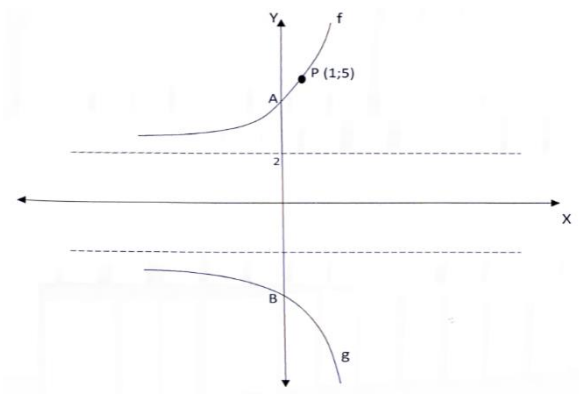
3.1.1	-10; -6; -2; … (add 4) Next to terms: 2; 6; …	√ 2;6 (1)
3.1.2	T_n = 4n + c -10 = 4(1) + c -14 = c ∴ T_n = 4n - 14	√ 4n √ - 14 (2)
3.1.3	S_6 = -10 - 6 – 2 + 2 + 6 + 10 = 0	√ correct terms added √ answer 0 (2)
3.1.4	4n – 14 = 78 4n = 92	√ = 78

	n = 23	√ n = 23 (2)
3.2.1	 Black tiles: 9; 18; 27; … White tiles: 16; 32 ;48; … ∴ Number of black tiles in 5th pattern = 9 X 5 = 45	√ 45 (1)
3.2.2	There will be (9 X 16) – (9 X 9) = 144 - 81 = 63 more white tiles	√ 144 - 81 √ 63 (2)

[10]

QUESTION 4

4.1 4.1.1	Given: $f(x) = \frac{3}{x} + 1$ and $g(x) = -\frac{1}{9}x^2 + 1$ Asymptotes f: Vertically x = 0 Horizontally y = 1	√ x = 0 √ y = 1 (2)
4.1.2	x-intercepts of g (y = 0) ∴ $-\frac{1}{9}x^2 + 1 = 0$ $-\frac{1}{9}x^2 = -1$ $x^2 = 9$ ∴ x = 3 or x = -3	√ $-\frac{1}{9}x^2 + 1 = 0$ √ **m diff of squares or square root both sides** √ x = 3 or x = -3 (3)
4.1.3	$f(x) = \frac{3}{x} + 1$ and $g(x) = -\frac{1}{9}x^2 + 1$  	√ shape and label f √ asymptote f √ **x-** intercept f √ **y-** intercept g √ **x-intercept** g √ **T.P. g** (6)
4.1.4	f is a decreasing function	√ decreasing

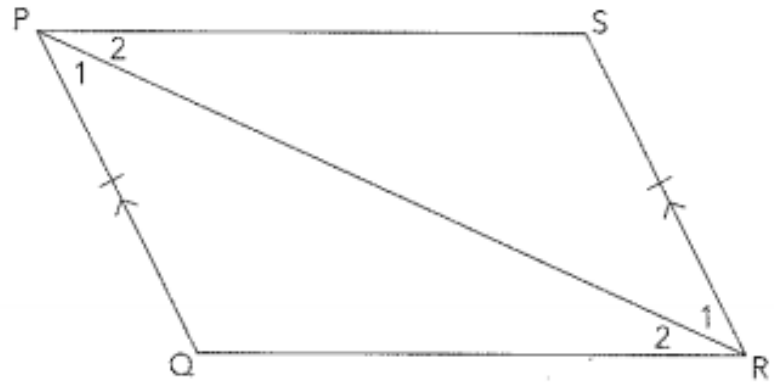
		(1)
4.1.5	Range g: $y \in (-\infty;1]$, $y \in R$ **OR:** $y \leq 1$, $y \in R$	√ answer (1)
4.1.6	Domain f: $x \in (-\infty;0) \cup (0;\infty)$, $x \in R$ **OR:** $x \neq 0$, $x \in R$	√ answer (1)
4.1.7	Equation of m through (-3;0) and (0;1) $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{0+3} = \frac{1}{3}$ $\therefore\ y = \frac{1}{3}x + 1$	√ $\frac{1}{3}$ √m substitution √ $y = \frac{1}{3}x + 1$ (3)
4.1.8	f(x) = m(x) $\therefore\ \frac{3}{x} + 1 = \frac{1}{3}x + 1$ $\therefore\ 9 + 3x = x^2 + 3x$ $x^2 = 9$ $x = 3$ (given: $x > 0$) $y = 2$ $\therefore$ P(3;2)	√ $\frac{3}{x} + 1 = \frac{1}{3}x + 1$ √ $x^2 = 9$ √ P(3;2) (3)
4.2.1	$f(x) = b^x + q$  Asymptote y = 2: $\therefore$ q = 2 $\therefore$ $y = b^x + 2$ (-1;5): $5 = b^1 + 2$ $\therefore$ b = 3	√ q = 2 √ replace (-1;5) √ b = 3 (3)
4.2.2	$g(x) = -f(x)$ $= -[3^x + 2]$ $= -(3^x) - 2$	√ $-(3^x)$ √ -2 (2)
4.2.3	y-intercept of f (x = 0): $y = 3^x + 2$ $\therefore\ y = 3^0 + 2$ $= 1 + 2$ $= 3$ $\therefore$ A(0;3)	√ 3 √ **A(0;3)** √ AB=6 (3)
4.2.4	f: $y = 3^x + 2$ for h: reflection about y-axis: $y = 3^{-x} + 2$ 3 units down: $y = 3^{-x} + 2 - 3$	√ 3^{-x} or (1/3)^x √ - 1 (2)

	$y = 3^{-x} - 1$	

[30]

QUESTION 5

In the diagram, PQ = RS and PQ||RS.



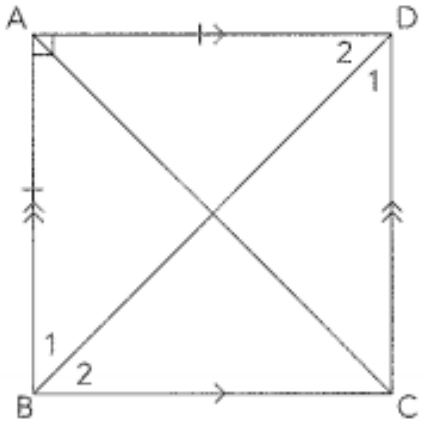
RTP: PQRS is a parm

PROOF: In ΔPQR and ΔRSP

1. PQ = SR	(given)	√
2. $\hat{P}_1 = \hat{R}_1$	(alt <'s PQ//SR)	√
3. PR is common		√ **(max 2 marks if layout incorrect)**
∴ $\Delta PQR \equiv \Delta RSP$	(SAS)	
4. ∴ PS = QR	($\equiv \Delta's$)	√
∴ PQRS is a parm	(Bth prs opp sides =)	√**conclusion** [5]

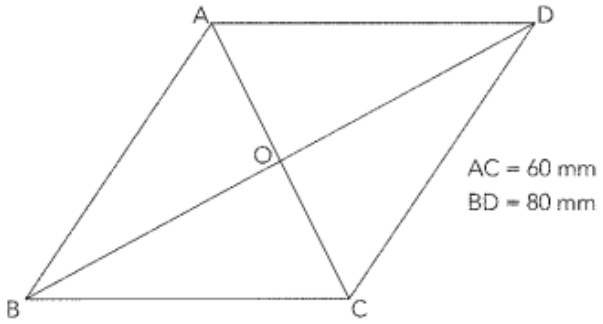
Question 6

6.1



	Statement	Reason
6.1.1	$\hat{B}_1 + \hat{D}_2 + B\hat{A}D = 180°$	< sum in Δ
6.1.2	$\hat{B}_1 = \hat{D}_2$	<'s opp =sides
6.1.3	$\hat{D}_1 = \hat{B}_1$	Alt <'s AB//DC
6.1.4	ABCD is a paralleogram	Bth prs opp sides //
6.1.5	ABCD is a square	Parm with 1 angle 90°

(5)

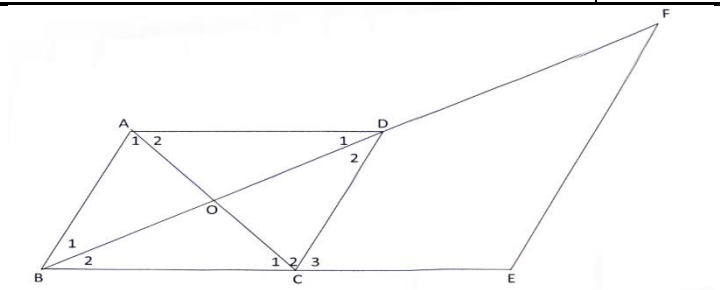


6.2.1	$A\hat{O}D = 90°$ (diags of rhombus)	√ 90 degrees √ reason (2)
6.2.2	AO = 30mm OD = 40mm AD = $\sqrt{30^2 + 40^2}$ (pythag) = 50mm	√ 30mm and 40mm √ statement √ reason √ 50mm (4)

[11]

QUESTION 7

7.1	Parallel and half the third side	√ parallel √ half (2)



7.2.1	DC ‖ EF line joining midpts of 2 sides of △, ‖ 3rd side DC ‖ AB opp sides rhombus ∴ AB ‖ EF	√ statement, reason √ statement, reason (2)
7.2.2	DC = ½EF midpt theorem ∴ AD = ½EF AD = DC ; adj sides rhombus	√ statement, reason √ statement, reason (2)
7.2.3	$\hat{F} = \hat{B}_1$ EF ‖ AB (from 6.1) , alt ∠'s = $\hat{D}_1 = \hat{B}_1$ ∠'s opp equal sides rhombus ∴ $\hat{D}_1 = \hat{F}$	√ statement, reason √ statement, reason (2)
7.2.4	True In ΔABD and ΔEBF $\hat{B}_1 = \hat{F}$ alt ∠'s = $\hat{D}_1 = B_2$ alt ∠'s = $\hat{A}_{1+2} = \hat{E}$ ∠'s of △ ∴ ΔABD ||| ΔEBF ∠∠∠	√ True √ statement, reason √ statement, reason √ statement, reason (4)
7.2.5	$\hat{B}_2 = \hat{F} = 35°$ given BE = EF $B\hat{O}C = 90°$ diag rhombus $\hat{C}_1 = 55°$ ∠'s △BOC	√ statement, reason √ statement, reason √ statement,reason (3)

[15]

GRADE 10 TERM 2 TOTAL 100

JUNE EXAMINATION 2017

	Taxonomy levels				Expressions Exponents	Patterns Equations Inequalities	Functions (Alg ; Trig)	Trig	Geom
	L1 20%	L2 35%	L3 30%	L4 15%					
Target	**20**	**35**	**30**	**15**					
1.1.1		2			√				
1.1.2		2			√				
1.2				4	√				
2.1.1		3				√			
2.1.2		3				√			
2.2.1		2				√			
2.2.2	2					√			
2.3			4			√			
2.4.1		1							
2.4.2	1								
2.4.3				2					
3.1.1	1					√			
3.1.2		2				√			
3.1.3			2			√			
3.1.4			2		√	√			
3.2.1	1					√			
3.2.2				2		√			
4.1.1	2						√		
4.1.2			3				√		
4.1.3			6				√		
4.1.4	1						√		
4.1.5	1						√		
4.1.6	1						√		
4.1.7			3				√		
4.1.8				3			√		
4.2.1			3				√		
4.2.2				2			√		
4.2.3		3					√		
4.2.4				2			√		
5.	5								√
6.1.1	1								√
6.1.2	1								√
6.1.3	1								√
6.1.4	1								√
6.2.1		2							√
6.2.2			4						√
7.1	2								√
7.2.1			2						√
7.2.2			2						√
7.2.3			2						

	7.2.4			4						√
	7.2.5				3					√
Total		18	20	44	18					