HILLCREST HIGH SCHOOL

Grade 12 Paper 1 Exam

June 2019

Examiner: Mrs Knight Moderator: Mr MacTavish

MARKS: 150 TIME: 3 hours

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions. Answer ALL the questions.
2. Write your name and **your Mathematics teacher**'s name on your answer booklet.
3. Clearly show ALL calculations, diagrams, graphs, etc which you have used to determine your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable) may be used, unless otherwise stated.
6. If necessary, answers should be rounded off to TWO decimal places, unless otherwise stated.
7. Number the answers EXACTLY as the questions are numbered.
8. Diagrams are not necessarily drawn to scale.
9. It is in your own interest to write legibly and to present your work neatly.
10. This question paper consists of 7 pages and 1 formula sheet.

Question 1

1.1 Solve for *x*:

1.1.1 $(x-3)(x+1)=0$ (2)

1.1.2 $\sqrt{x^3}=512$ (3)

1.1.3 $x(x-4)<0$ (2)

1.1.4 $3^x(3^x-9)=0$ (2)

1.2 Given: $f(x)=x^2-5x+2$

1.2.1 Solve for *x* if *f(x)* = 0, to two decimal places (3)

1.2.2 For which values of *c* will *f(x)* = *c* have no real roots? (4)

1.3 Solve for *x* and *y*:

$x=2y+2$

$x^2-2xy+3y^2=4$ (6)

1.4 Calculate the maximum value of S if S = $\frac{6}{x^2+2}$ (2)

[24]

Question 2

2.1 Prove that in an arithmetic series of which the first term is *a* and where the constant difference is *d*, the sum of the first *n* terms is given by $S_n=\frac{n}{2}[2a+(n-1)d]$. (4)

2.2 Given the following sequence:

-5; -1; 3; 7; …; 35

2.2.1 Determine the number of terms in the sequence. (3)

2.2.2 Calculate the sum of the sequence. (2)

2.3 For an arithmetic series, consisting of 15 terms, $S_n=2n-n^2$.

2.3.1 Determine the first term of the sequence. (2)

2.3.2 The sum of the last 3 terms. (3)

[14]

Question 3

3.1 A quadratic number pattern $T_n = an^2 + bn + c$ has a fourth term equal to -5 (i.e. $T_4 = -5$), while the first differences of the quadratic sequence are given by: -12; -8; -4

3.1.1 Write down the values of the first three terms of the quadratic sequence. (3)

3.1.2 Calculate the value of the n^{th} term of the sequence. (4)

3.2 Consider the geometric series:

$4 + p + \frac{p^2}{4} + \frac{p^3}{16} + \cdots$

3.2.1 Calculate the value(s) of *p* for which the series converges. (3)

3.2.2 Calculate the value of *p* if the sum to infinity is 3. (3)

[13]

Question 4

Given: $f(x) = \frac{2}{x+1} - 3$

4.1 Calculate the coordinates of the y-intercept of *f*. (2)

4.2 Calculate the coordinates of the x-intercept of *f*. (3)

4.3 Sketch the graph of *f* in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (3)

4.4 One of the axes of symmetry of *f* is a decreasing function. Write down the equation of this axis of symmetry. (3)

[11]

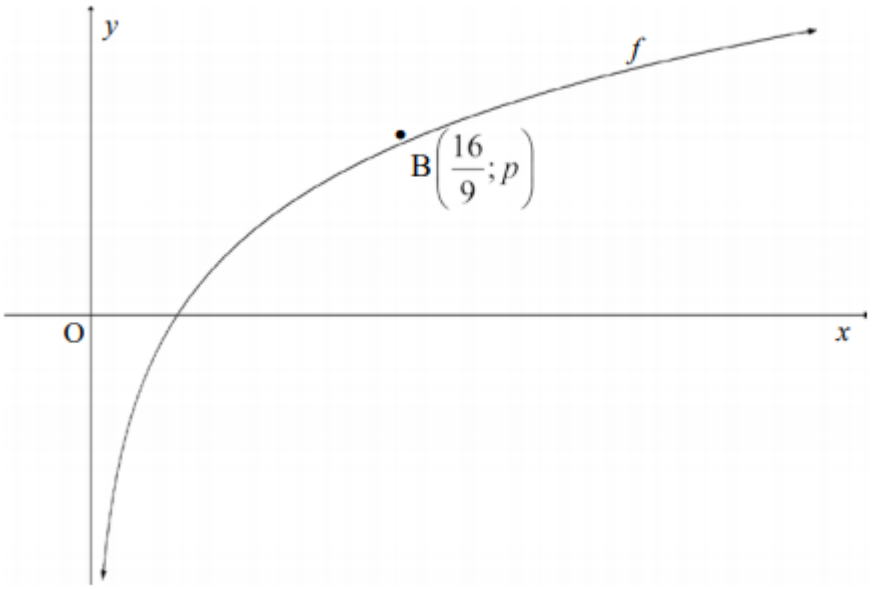
Question 5

5.1 The graph of an increasing exponential function with equation $g(x) = a.b^x + q$ has the following properties:

- Range: y > -3
- The points (0 ; -2) and (1 ; -1) lie on the graph of *g*.

Determine the equation that defines *g*. (4)

5.2 The graph of $f(x) = log_{\frac{4}{3}}x$ is drawn below. B$(\frac{16}{9}; p)$ is a point on *f*.



5.2.1 For which value(s) of *x* is $log_{\frac{4}{3}}x \leq 0$? (2)

5.2.2 Determine the value of p, without the use of a calculator. (3)

5.2.3 Write down the equation of the inverse of *f* in the form y = … (2)

5.2.4 Write down the range of $y = f^{-1}(x)$ (2)

5.2.5 The function $h(x) = \left(\frac{3}{4}\right)^x$ is obtained after applying two reflections on *f*.

Write down the coordinates of B'', the image of *B* on *h*. (2)

[15]

Question 6

The graph of *g* is defined by the equation $g(x) = \pm\sqrt{ax}$. The point (8; 4) lies on *g*.

6.1 Show that *a* = 2. (1)

6.2 If $g(x) \geq 0$, for what values of *x* will *g* be defined? (1)

6.3 If $h(x) = x - 4$ is drawn, determine ALGEBRAICALLY the point(s) of intersection of *h* and *g*. (5)

6.4 Hence, determine the values of *x* for which $g(x) > h(x)$. (2)

[9]

Question 7

7.1 Determine $f'(x)$ from first principles if $f(x) = 3x^2 - 5$ (5)

7.2 Determine $\frac{dy}{dx}$ if:

7.2.1 $y = 2x^5 + \frac{4}{x^3}$ (3)

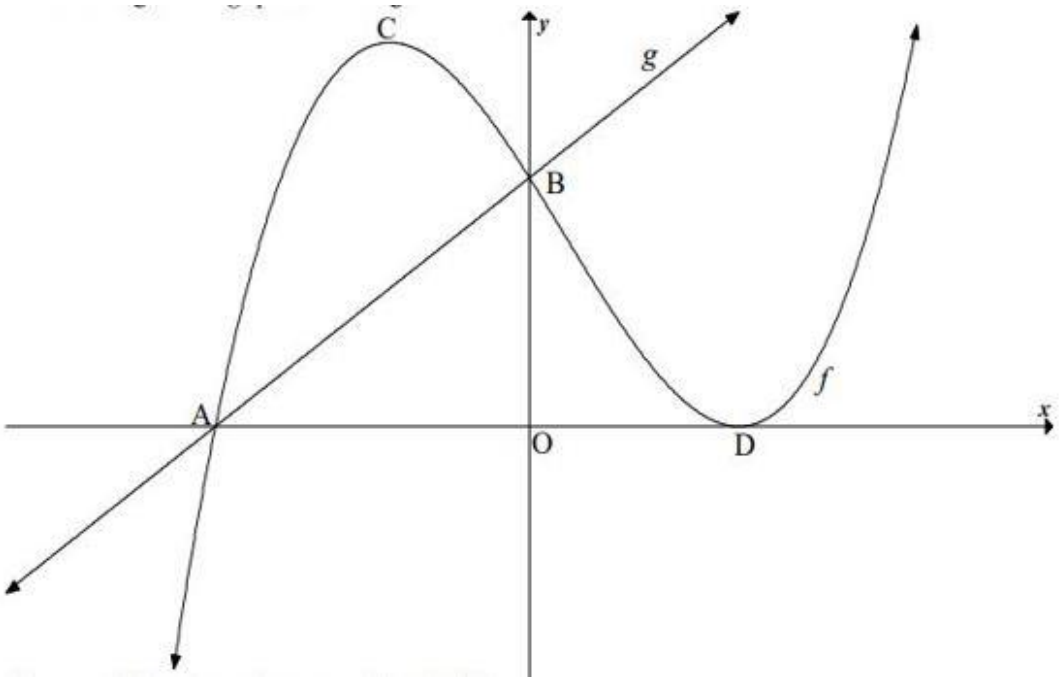
7.2.2 $y = \left(\sqrt{x} - x^2\right)^2$ (4)

[12]

Question 8

Sketched below are the graphs of $f(x) = (x-2)^2(x-k)$ and $g(x) = mx + 12$

- A and D are the *x*-intercepts of *f*.
- B is the common *y*-intercept of *f* and *g.*
- C and D are turning points of *f.*
- The straight line *g* passes through A.



8.1 Write down the y-coordinate of B. (1)

8.2 Calculate the x-coordinate of A. (3)

8.3 If *k* = -3, calculate the coordinates of C. (6)

8.4 For which values of *x* will *f* be concave down? (3)

8.5 If $p(x) = x^3 + 2x$

Show, using relevant calculations, why it is not possible for a tangent drawn to the graph of p to have a negative gradient. (3)

[16]

Question 9

The mass of a baby in the first 30 days of life is given by

$M(t) = t^3 - 9t^2 + 3000 \qquad 0 \leq t \leq 30$

t is the time in days and *M* is the mass of the baby in grams.

9.1 Write down the mass of the baby at birth. (1)

9.2 A baby's mass usually decreases in the first few days after birth. On which day will the baby's mass return to its birth mass? (4)

9.3 On which day will this baby have a minimum mass? (4)

9.4 On which day will the baby's mass be decreasing the fastest? (2)

[11]

Question 10

10.1 Calculate the effective interest rate per annum if the nominal interest rate is 15% compounded monthly. (3)

10.2 Thembalethu applied for a loan of R75 000 from the bank at 12% simple interest per annum for 8 years.

10.2.1 Calculate Thembalethu's monthly instalment. (3)

10.2.2 The bank wants to change the interest to a compound interest per annum without affecting Thembalethu's monthly payment. Calculate the compound interest rate that the bank would have to charge, correct to 2 decimal places. (4)

10.3 R100 000 is invested in account which offers interest at 8% compounded quarterly for the first 18 months. Thereafter the interest rate changes to 6% p.a. compounded monthly. Three years after the initial investment, R12 000 is withdrawn from the account. How much will be in the account at the end of 5 years? (4)

[14]

Question 11

11.1 Given: $P(A) = 0{,}55$

$P(B) = 0{,}35$

$P(A \text{ or } B) = 0{,}8$

Determine whether the events A and B are independent or not. Show all relevant calculations used in determining the answer. (4)

11.2 In a particular class at Hillcrest High School all the learners take Mathematics and Physical Science.

The probability that a randomly selected learner from that class will pass:

- Mathematics is 0,6
- Physical Science is 0,7
- both of these subjects is *x*; and
- neither of these subjects is 0,2.

11.2.1 Draw a Venn diagram to illustrate the above information (2)

11.2.2 Calculate *x.* (2)

11.2.3 Determine the probability that a randomly selected learner from that class

a) will pass at least one of the two subjects (1)

b) will pass exactly one of the two subjects (2)

[11]