HILLCREST HIGH SCHOOL

GRADE 12

MATHEMATICS PAPER 2

JUNE 2019

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.

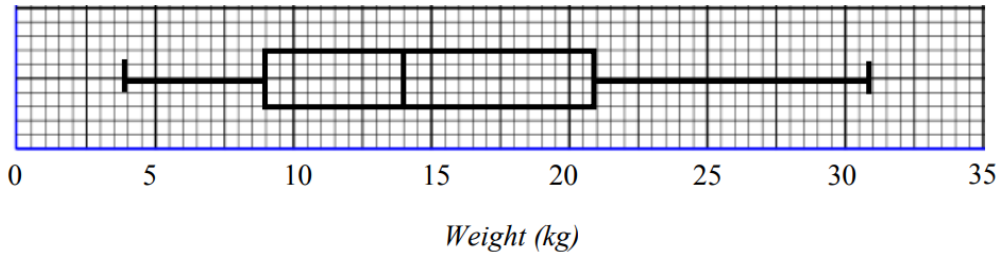
MARKS:	**150**	**EXAMINER:**	**MR. GA MAC TAVISH**
TIME:	**3 hours**	**MODERATOR:**	**MRS. A SPARKS**

This question paper consists of 10 pages and 1 information sheet.

QUESTION 1

1.1 South African Airways (SAA) conducted a survey on the weight of luggage (in kilograms) customers check-in before their flight to Beijing.

They have summarised the data in the box and whisker diagram below.



1.1.1 Determine:

1.1.1.1 the five number summary of the data. (2)

1.1.1.2 the interquartile range (1)

1.1.1.3 the number of bags with a weight of 21 kg or more, if it is given that 220 bags were on the flight. (2)

1.1.2 It is further given that the mean ($\bar{x}$) for the above data is 16kg.

Hence, or otherwise, comment on the skewness of the data. Justify your answer. (2)

1.2 The heights (h cm) of 270 SAA employees are measured for their new uniforms and the results are shown in the table below.

Interval	Frequency
$120 < h \leq 130$	15
$130 < h \leq 140$	24
$140 < h \leq 150$	36
$150 < h \leq 160$	45
$160 < h \leq 170$	50
$170 < h \leq 180$	43
$180 < h \leq 190$	37
$190 < h \leq 200$	20

1.2.1 Complete the cumulative frequency column in the table provided in the SPECIAL ANSWER BOOK. (2)

1.2.2 Write down the modal class. (1)

1.2.3 Draw an ogive (cumulative frequency graph) to represent the data on the grid given in the SPECIAL ANSWER BOOK. (3)

1.3 In Beijing, the midday temperatures, in $°C$, are recorded during a week in December.

-3	0	-2	-2	1	x	-1

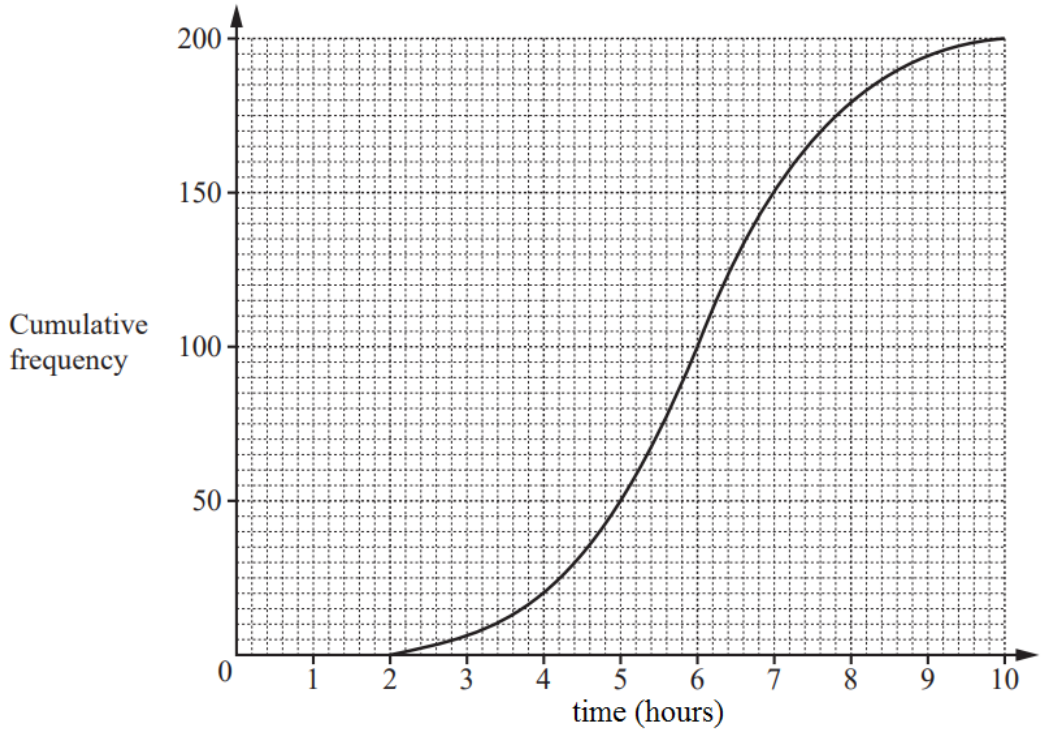
1.3.1 Determine the mean in terms of x. (1)

1.3.2 If it is given that the mean $(\bar{x})$ is $-1\,°C$, determine the value of x. Show ALL calculations. (2)

1.3.3 Determine the standard deviation (σ), if it is given that $x = 0\,°C$. (1)

1.4 A survey was conducted amongst visitors at the Happy Valley amusement park in Beijing. Visitors were asked about the number of hours they had spent, on a particular day, at the amusement park.
The results of the survey are shown in the cumulative frequency graph below.

Cumulative frequency graph of the number of hours spent by visitors at the Happy Valley amusement park.



Use the graph to determine:

1.4.1 how many visitors participated in this survey (1)

1.4.2 the interquartile range (2)

1.4.3 the percentage of visitors that visited the amusement park for more than 9 hours (2)

[22]

QUESTION 2

2.1 $M(1\,;\,b)$ is the midpoint of the line segment joining $A(a\,;\,4)$ and $B(5\,;\,6)$.

Determine the values of a and b. (3)

2.2 The points $C(1\,;-2)$, $D(5\,;\,1)$ and $E(c^2\,;\,c+1)$ are collinear.

Determine the value(s) of c. (5)

2.3 The lines $2y+7x=5$ and $4x+dy+3=0$ are perpendicular to each other.

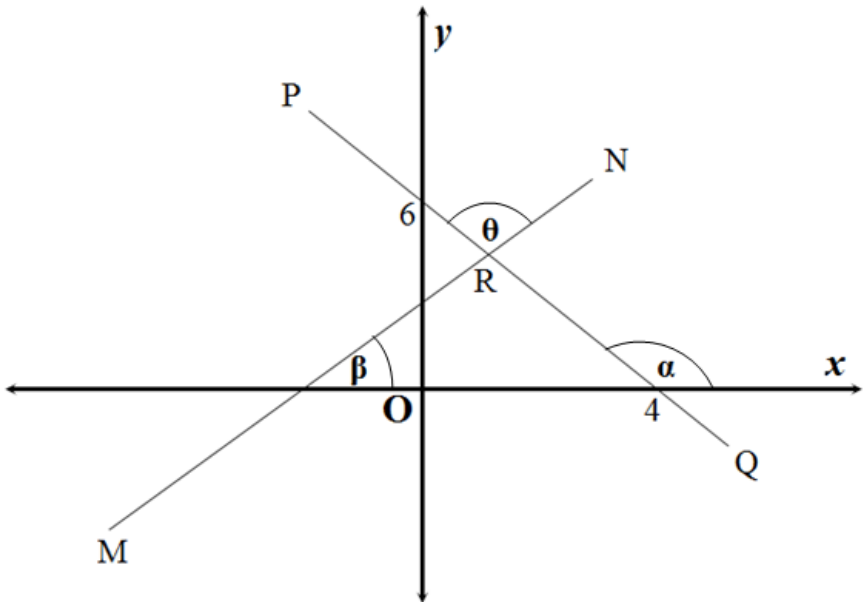
Find the value of d. (5)

[13]

QUESTION 3

In the figure below line PQ cuts the *y*-axis at 6 and the *x*-axis at 4. Line MN has the equation $y=x+3$. The angles of inclination of line PQ and MN are α and β respectively.

The two lines intersect at R in the first quadrant making an angle of θ $\left(P\hat{R}N=\theta\right)$.



Determine:

3.1 the size of α (3)

3.2 the size of β (1)

3.3 the size of θ (1)

3.4 the coordinates of R, the point of intersection of PQ and MN. (4)

[9]

QUESTION 4

4.1 Write down the equation of the circle with a centre $(-2;3)$ and a radius of $\sqrt{13}$ units. (2)

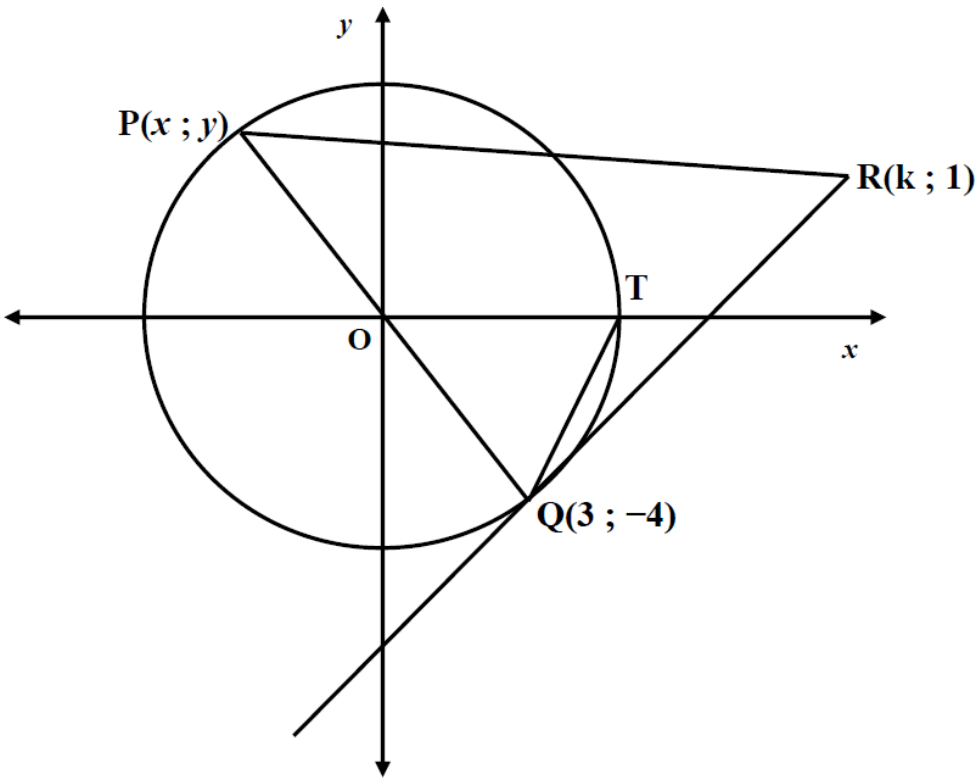
4.2 $x^2 + y^2 + 4x - 12y + 4 = 0$ is the equation of a circle with centre M and a radius r.

4.2.1 Determine the coordinates of the centre M and the length of the radius r. (4)

4.2.2 Calculate the coordinates of the point(s) where this circle intersects the x-axis. (2)

4.2.3 Determine the equation(s) of the tangent(s) to this circle which are parallel to the y-axis. (2)

4.3 In the figure below, the origin O is the centre of the circle. P(x ; y) and Q(3 ; −4) are two points on the circle and POQ is a straight line. R is the point (k ; 1) and RQ is a tangent to the circle. T is an x-intercept of the circle.



Determine:

4.3.1 the length of TQ. (Leave your answer in simplified surd form.) (4)

4.3.2 the equation of RQ in the for $y = mx+c$. (4)

4.3.3 the value of k. (2)

[20]

QUESTION 5

5.1 If $\tan\alpha = -\frac{5}{12}$ and $0° \leq \alpha \leq 180°$, use a sketch to determine the value of the following expression (WITHOUT USING A CALCULATOR):

$3\sin\alpha - 2\cos\alpha$ (4)

5.2 Simplify the following expression to ONE trigonometric ratio of θ :

$$\frac{\sin(360° - \theta) + \cos 270°}{\sin(90° - \theta) + \tan(180° + \theta) + \tan(360° - \theta)}$$ (6)

5.3 Simplify the following, WITHOUT THE USE OF A CALCULATOR.

$$\frac{\cos 170°.\cos 30° + \cos 280°.\sin 30°}{\sin 25°.\cos 25°}$$ (6)

[16]

QUESTION 6

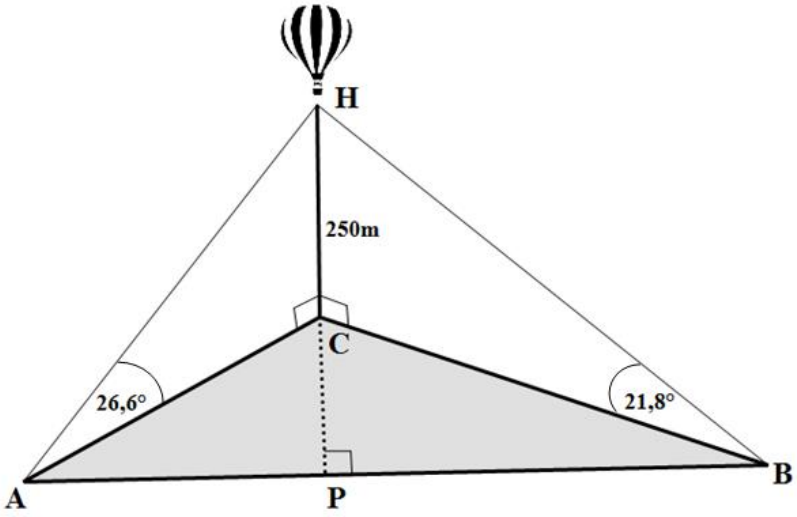
6.1 Prove that: $\cos(x + y) - \cos(x - y) = -2\sin x.\sin y$ (2)

6.2 Hence, or otherwise, prove that: $\frac{\cos 3x - \cos x}{-4\sin^2 x.\cos x} = 1$ (5)

[7]

QUESTION 7

Two observers at A and B sight a hot air balloon H hovering at 250 m directly above C. The angles of elevation from A and B to H respectively are 26,6° and 21,8°.
A, B and C are in the same horizontal plane.



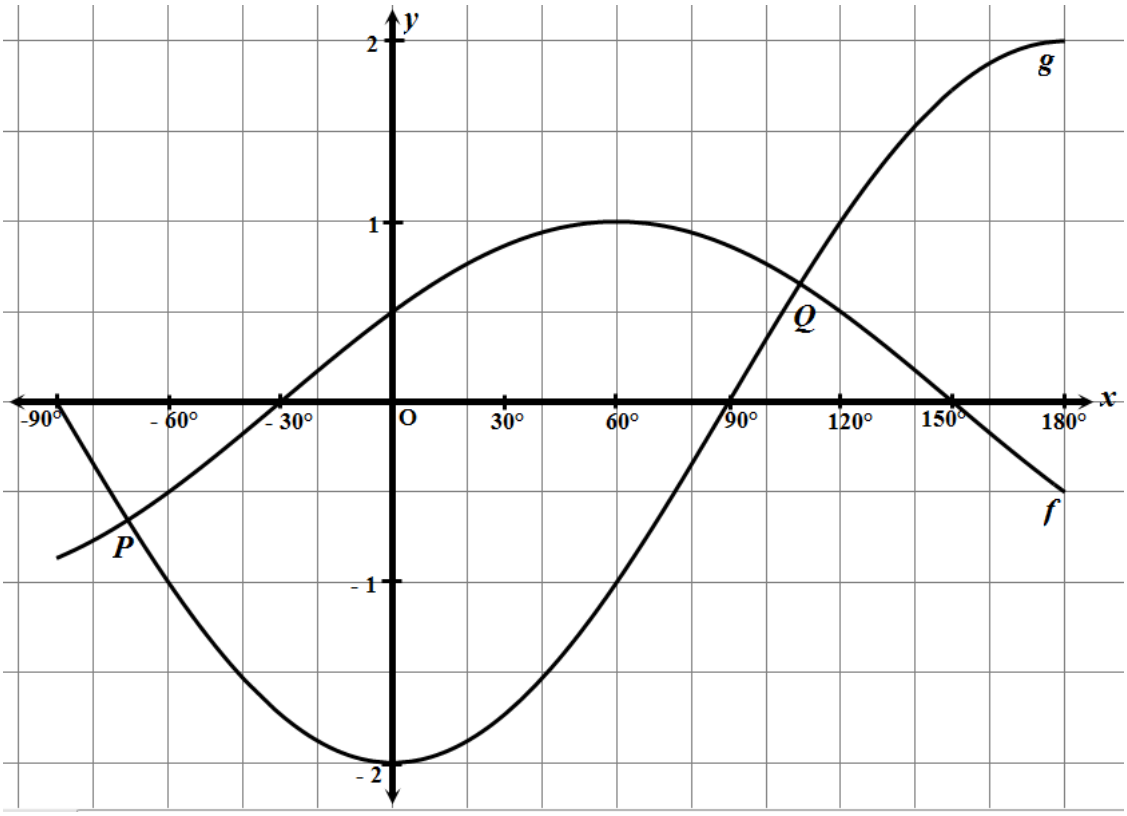
7.1 Calculate the distance between the observers A and B if $A\hat{C}B = 104,5°$ (6)

7.2 Calculate the size of $C\hat{A}B$ (3)

[9]

QUESTION 8

The graphs of $f(x)=\sin(x+30°)$ and $g(x)=-2\cos x$ for $-90° \le x \le 180°$ are given below. The graphs intersect at point P and point Q.



8.1 Use the graph to calculate $f(0)-g(0)$. (1)

8.2 Determine the period of g. (1)

8.3 The graphs of $f(x)$ and $g(x)$ intersect when $f(x) = g(x)$, at P and Q.

8.3.1 Expand the following compound angle: sin (A + B) = ……. (1)

8.3.2 Hence, or otherwise, calculate the x-coordinates of P and Q. (6)

8.4 Write down the values for which $f(x) \ge g(x)$. (2)

8.5 If graph h is a transformation of f such that $h(x)=-f(x-60°)$.

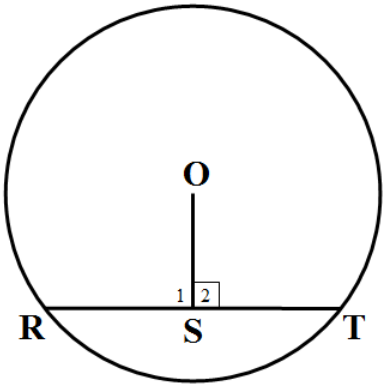
Describe, in words, the transformation from f to h. (2)

[13]

Give reasons for your statements in QUESTION 9 and 10.

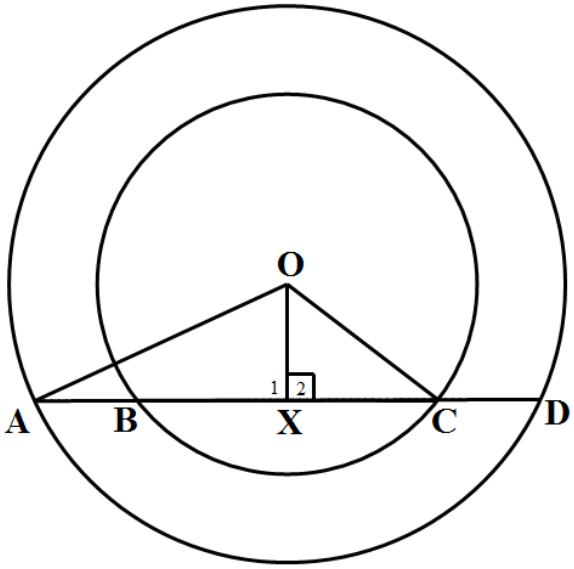
QUESTION 9

9.1 In the diagram below, O is the centre of the circle and OS is perpendicular to the chord RT.



Prove, using Euclidian geometry methods, the theorem that states that RS = ST. (5)

9.2 The line ABCD intersects two concentric circles with centre O as shown in the diagram below. OA = 25 cm and OC = 17 cm.



9.2.1 Determine AC if OX = 15 cm. (5)

9.2.2 Show that AB = CD. (3)
[13]

QUESTION 10

10.1 Complete the statements of the following theorems by writing down only the missing word(s) in each case:

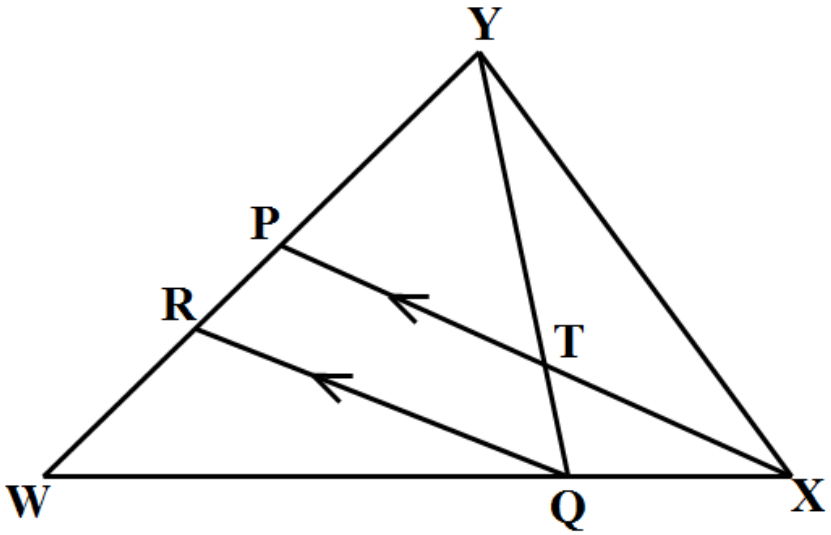
10.1.1 The opposite angles of a cyclic quadrilateral are ... (1)

10.1.2 If two triangles are equiangular, the corresponding sides are ... (1)

10.2 In the diagram below, XP is the **perpendicular bisector** of side WY of ΔWXY.

Q is a point on WX such that WQ : WX = 3 : 5. XP and YQ intersect at T.

QR is drawn parallel to XP.



Determine:

10.2.1 $\frac{YP}{YR}$ (3)

10.2.2 $\frac{QR}{TP}$ (2)

10.2.3 $\frac{\text{Area of } \Delta TPY}{\text{Area of } \Delta QRY}$ (3)

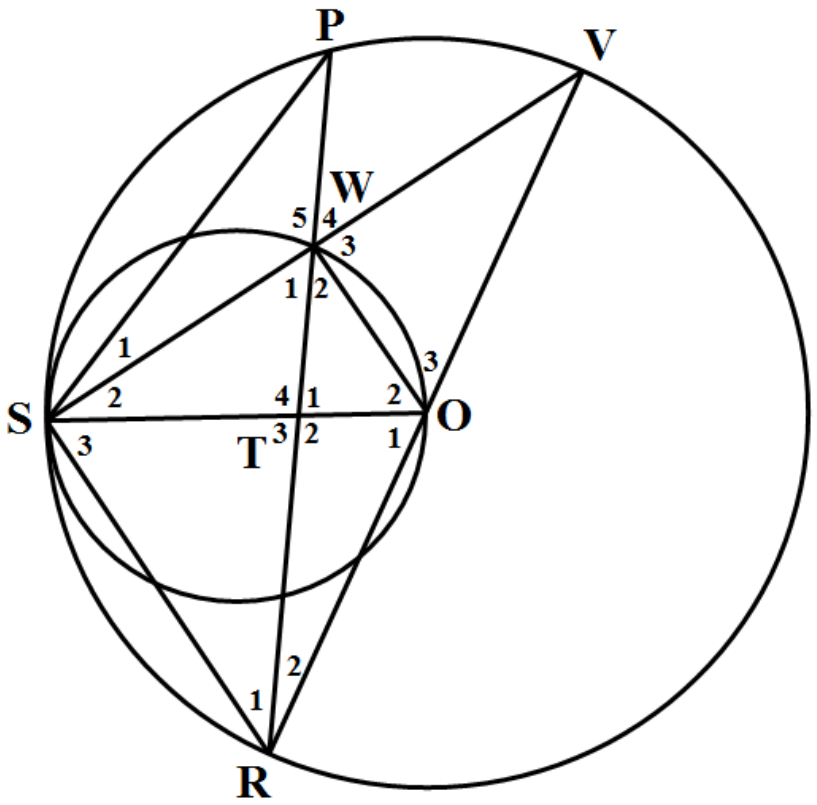
10.3 In the diagram below, two circles touch internally at S.

O is the centre of the bigger circle and OS is a diameter of the smaller circle.

PR is a double chord such that PT = TR intersecting the smaller circle at W.

SW is produced to meet the bigger circle at V. VOR is a straight line.

WO and PS are drawn.



Prove that:

10.3.1 SW = WV (3)

10.3.2 $SW^2 = PW.WR$ (6)

10.3.3 $SW^2 = WT.WR$ (5)

10.3.4 PW : WR = 1 : 3 (4)

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TOTAL: 150

GRADE 12 Examination

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x\left[\left(1+i\right)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right) \qquad m = \tan\theta$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\left(x-a\right)^2 + \left(y-b\right)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$