HILLCREST HIGH SCHOOL

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Mathematics Department
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GRADE 12

MATHEMATICS JUNE EXAM

Marking Guideline

This question paper consists of 6 pages and 1 information sheet.

QUESTION 1

Given the following sequence: 7 ; 10 ; 13 ; 16; …

1.1 Is the sequence arithmetic, quadratic or geometric? Give a reason for answer. (1)

1.2 Calculate the value of T_{100} . (2)

1.3 Calculate the value of n if $S_n = 5730$. (4)

1.4 If 5 is added to each term, calculate S_{30} of the new sequence thus formed. (3)

[10]

QUESTION 2

Consider the geometric series: 18 + 6 + 2 + …

2.1 Write down the value of the constant ratio. (1)

2.2 Calculate the value of T_{12}. (2)

2.3 Evaluate: $\sum_{n=8}^{\infty} 18(3)^{1-n}$ (3)

[6]

QUESTION 3

Determine values of x for which the sequence below will converge:

$$1-3x \ ; \ \frac{-(1-3x)^2}{3} \ ; \ \frac{(1-3x)^3}{9} \ ; \ ....$$ **[4]**

QUESTION 4

Given $f(x) = -\log_3 x$ and $g(x) = \frac{x}{3} + 1$.

4.1 Determine $f^{-1}(x)$ and $g^{-1}(x)$, writing each in the form $y = ...$ (4)

4.2 State the domain and range for each inverse function. (4)

[8]

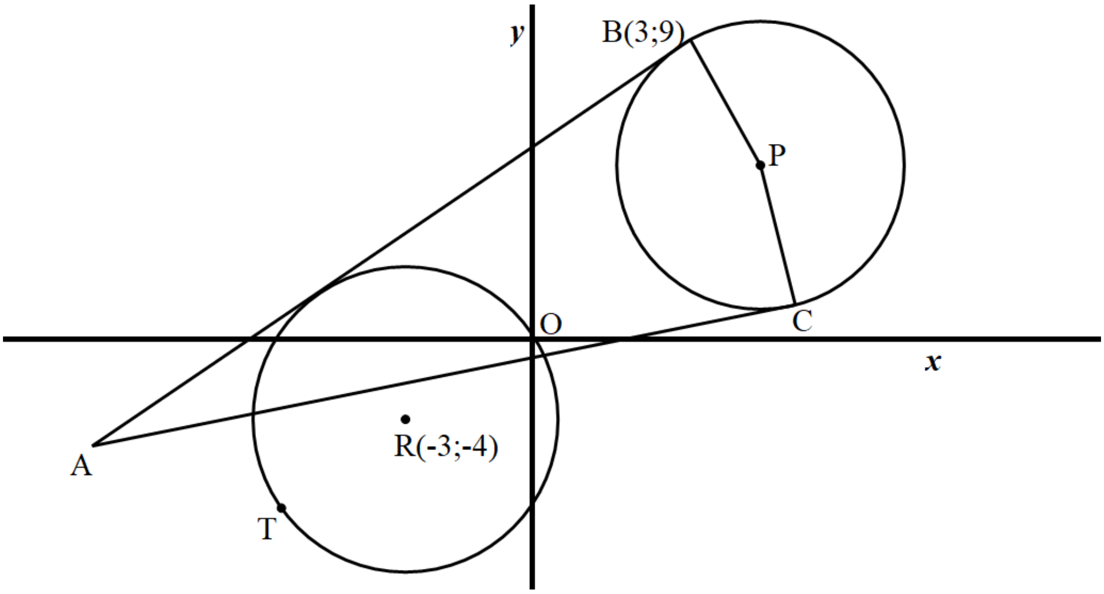
QUESTION 5

Given circle centre P with equation $x^2 - 12x + y^2 - 12y + 54 = 0$.

AB and AC are both tangents to the circle with centre P.

AB touches the circle at B(3;9).

A second circle is drawn with its centre at R(– 3; –4) and passing through the origin (O).



5.1 Determine the equation of the circle with centre R. (3)

5.2 Determine the co-ordinates of T, if OT is a diameter through R. (2)

5.3 Calculate the co-ordinates of P, the centre of the other circle. (4)

5.4 Determine the equation of AB. (5)

5.5 What shape is BACP? Give a reason for your answer. (2)

5.6 Does the point F(5;4) lie inside, outside or on the circle with centre P? Show all calculations. (4)

[20]

QUESTION 6

6.1 If $5\sin A + 2 = 0$ and $0^\circ \leq A \leq 270^\circ$, use a sketch to calculate the value of:

6.1.1 cos A (2)

6.1.2 cos 2A (3)

6.2 Simplify each of the following, without the use of a calculator:

6.2.1 $\dfrac{\sin 850^\circ}{\cos 140^\circ . \cos^2 315^\circ}$ (5)

6.2.2 $\dfrac{\cos 15^\circ \cos 105^\circ}{\cos(45^\circ - x)\cos x - \sin(45^\circ - x)\sin x}$ (6)

6.3 Prove the following identity:

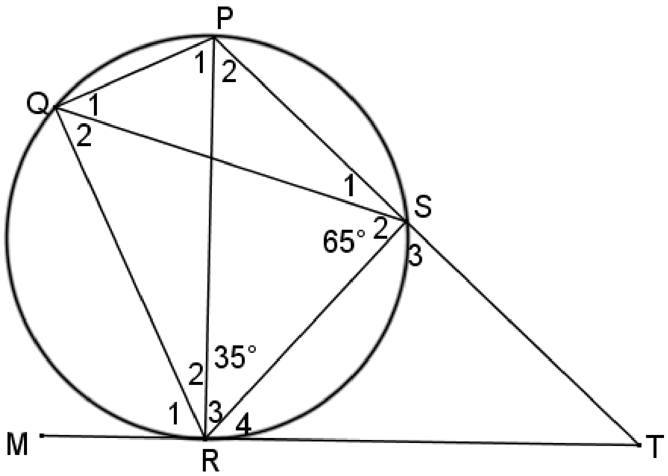
$$\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$$ (5)

6.4 If $\cos 55^\circ = p$ determine the value of $\cos 5^\circ$ in terms of *p*. (5)

[26]

QUESTION 7

7.1 In the sketch below P, Q, R and S are points on the circle. PR is the diameter of the circle and MRT is a tangent at R.
$\hat{S}_2 = 65^\circ$ and $\hat{R}_3 = 35^\circ$



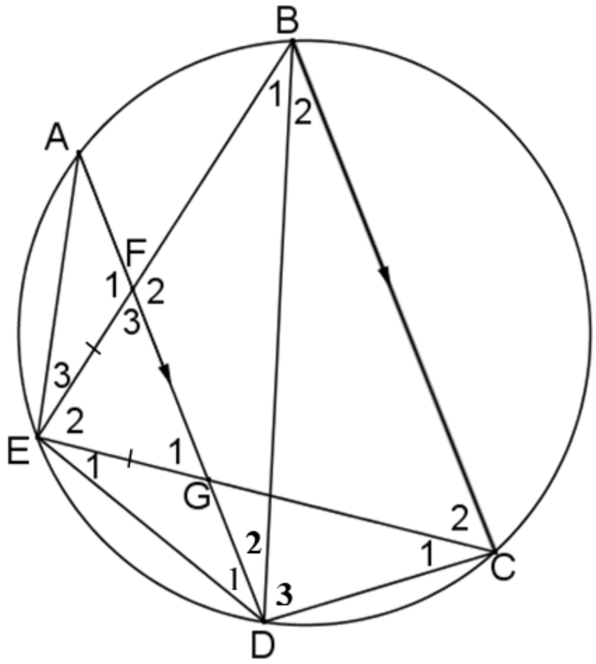
Determine, with reasons, the sizes of:

7.1.1 $\hat{R}_1$ (2)

7.1.2 $\hat{R}_4$ (2)

7.1.3 $\hat{S}_3$ (2)

7.2 In the sketch below AD || BC with A, B, C, D and E points on the circumference of the circle. EB, EC and BD are straight lines and EF = EG.



Prove, with reasons, that:

7.2.1 $\hat{B}_2 = \hat{D}_2$ (2)

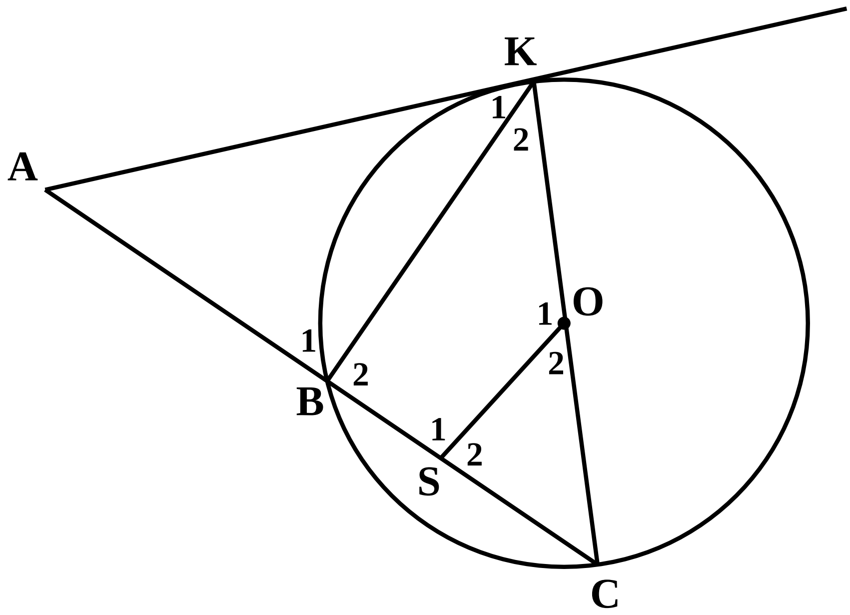
7.2.2 $\hat{E}_1 = \hat{E}_3$ (3)

7.2.3 $\Delta AEF \;|||\; \Delta CED$ (3)

[14]

QUESTION 8

8.1 In the diagram below:
KOC is a diameter of circle with centre O. KA is a tangent to the circle at point K.
AC intersects the circle at B. BS = SC.
KB and OS are joined.

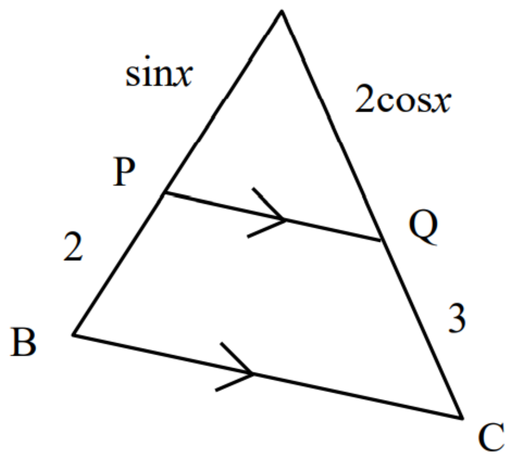


Prove that:

8.1.1 $\Delta COS\ ///\ \Delta KAB$ (4)

8.1.2 $2SO^2 = CS.BA$ (4)

8.2 In the triangle below, PQ // BC. $AP = \sin x$, $PB = 2$, $AQ = 2\cos x$, $QC = 3$.



Determine the values of x for which the diagram is valid and where $x \in [0° ; 360°]$ (4)

[12]

TOTAL: 100

INFORMATION SHEET: MATHEMATICS

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$A=P(1+ni)$ $\quad A=P(1-ni)$ $\quad A=P(1-i)^n$ $\quad A=P(1+i)^n$

$T_n=a+(n-1)d$ $\quad S_n=\frac{n}{2}\{2a+(n-1)d\}$

$T_n=ar^{n-1}$ $\quad S_n=\frac{a\left(r^n-1\right)}{r-1}\ ;\ r\neq 1$ $\quad S_\infty=\frac{a}{1-r}\ ;\ -1<r<1$

$F=\frac{x\left[(1+i)^n-1\right]}{i}$ $\quad P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad M\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $\quad m=\tan\theta$

$y=mx+c$ $\quad y-y_1=m(x-x_1)$ $\quad m=\frac{y_2-y_1}{x_2-x_1}$

$(x-a)^2+(y-b)^2=r^2$

In ΔABC: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$a^2=b^2+c^2-2bc.\cos A$

$\text{area}\ \Delta ABC=\frac{1}{2}ab.\sin C$

$\sin(\alpha+\beta)=\sin\alpha.\cos\beta+\cos\alpha.\sin\beta$ $\quad \sin(\alpha-\beta)=\sin\alpha.\cos\beta-\cos\alpha.\sin\beta$

$\cos(\alpha+\beta)=\cos\alpha.\cos\beta-\sin\alpha.\sin\beta$ $\quad \cos(\alpha-\beta)=\cos\alpha.\cos\beta+\sin\alpha.\sin\beta$

$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\1-2\sin^2\alpha\\2\cos^2\alpha-1\end{cases}$ $\quad \sin 2\alpha=2\sin\alpha.\cos\alpha$

$\bar{x}=\frac{\sum fx}{n}$ $\quad \sigma^2=\frac{\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}{n}$

$P(A)=\frac{n(A)}{n(S)}$ $\quad P(A\ or\ B)=P(A)+P(B)\ -P(A\ and\ B)$

$\hat{y} = a + bx$ $$b = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x-\bar{x})^2}$$