HILLCREST HIGH SCHOOL

HILLCREST HIGH SCHOOL
INTERNAL ASSESSMENT

GRADE 12

MATHEMATICS
Paper 1
JUNE 2022

MARKS: **150**

TIME: **3 hours**

This question paper consists of 9 pages and 1 information sheet.

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INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **10** questions.
2. Read the questions carefully.
3. Answer ALL the questions.
4. Number your answers exactly as the questions are numbered.
5. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
6. Answers only will NOT necessarily be awarded full marks.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
9. Diagrams are NOT necessarily drawn to scale.
10. Write neatly and legibly.
11. An information sheet with formulae is included at the end of the question paper.

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QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 6 = x$ (3)

1.1.2 $x(x+2) - 4 = 2$ (correct to TWO decimal places) (3)

1.1.3 $-x^2 - 2x + 8 > 0$ (3)

1.1.4 $3^{2x+1} + 8.3^x - 3 = 0$ (4)

1.2 Solve the following equations simultaneously:

$x - 2y = 2$ and $y - 3 = (x+1)(x-3)$. (6)

1.3 If $x = \sqrt{3 - 2\sqrt{2}}$ and $y = \sqrt{2} - 1$, show that $x = y$. (5)

[24]

QUESTION 2

2.1 Given the arithmetic series: $-12 - 8 - 4 ..... + 164$

2.1.1 How many terms are there in the series? (4)

2.1.2 Calculate the sum of the series. (3)

2.1.3 Write the above series in sigma notation. (2)

2.2 Given the geometric series: $5(3)^4 + 5(3)^3 + 5(3)^2 .....$

2.2.1 Explain why the series converges. (2)

2.2.2 Determine the 8^{th} term of the series. (3)

2.2.3 Calculate the sum to infinity of the series. (2)

2.3 The sum of the first n terms of an arithmetic series is: $S_n = \frac{3n^2 - n}{2}$.

2.3.1 Determine S_{10}. (1)

2.3.2 How many terms would have a sum of 590? Show all calculations. (4)

[21]

QUESTION 3

A quadratic pattern has the following: $T_2 = 1$, $T_3 = 0$ and $T_5 = -8$.

3.1 Determine the value of the common second difference of this sequence. (5)

3.2 Find the value of T_1. (2)

[7]

QUESTION 4

Given: $f(x) = \frac{2}{x+3} + 1$

4.1 Write down the equations of the asymptotes of f . (2)

4.2 Calculate where f intersects both the x and y axes. (3)

4.3 Sketch the graph of f showing clearly your answers to QUESTIONS 4.1 and 4.2. (3)

4.4 Write down the domain of f . (1)

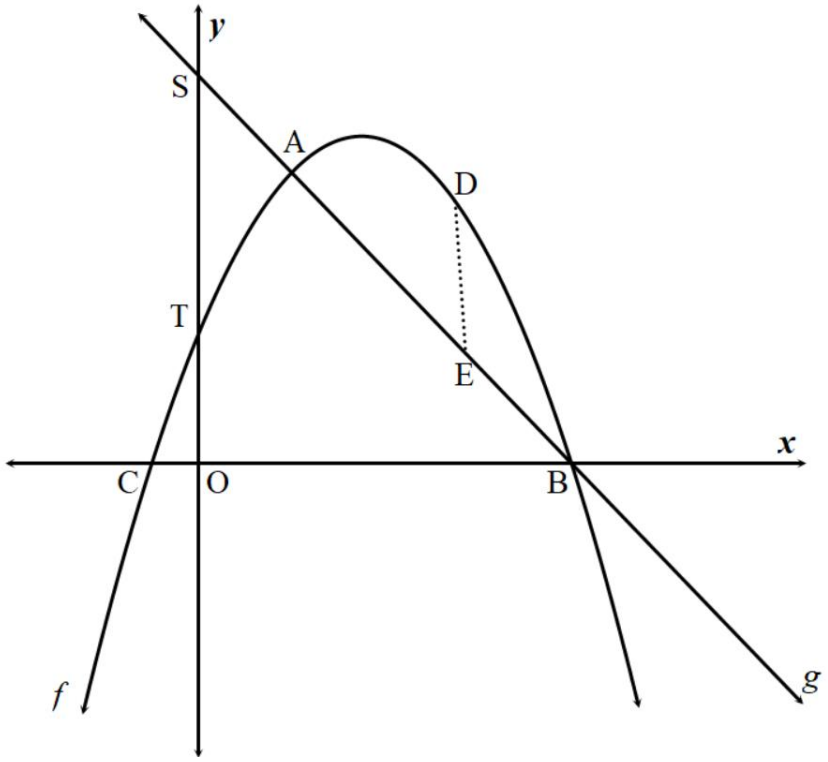
4.5 Determine the equation of the decreasing axis of symmetry. (2)

4.6 Write down the coordinates of the point of intersection of the two axes of symmetry. (2)

[13]

QUESTION 5

The graphs of $f(x)=-x^2+7x+8$ and $g(x)=-3x+24$ are sketched below.
f intersects the axes at C,T and B while g intersects the axes at S and B.
The functions f and g intersect at A and B.



5.1 Write down the length of ST. (1)

5.2 Calculate the length of CB. (3)

5.3 D is a point on the graph of f between A and B with $DE \parallel ST$.
Determine the maximum length of DE. (4)

5.4 Determine the coordinates of the turning point of h if $h(x)=f(x-1)+2$. (3)

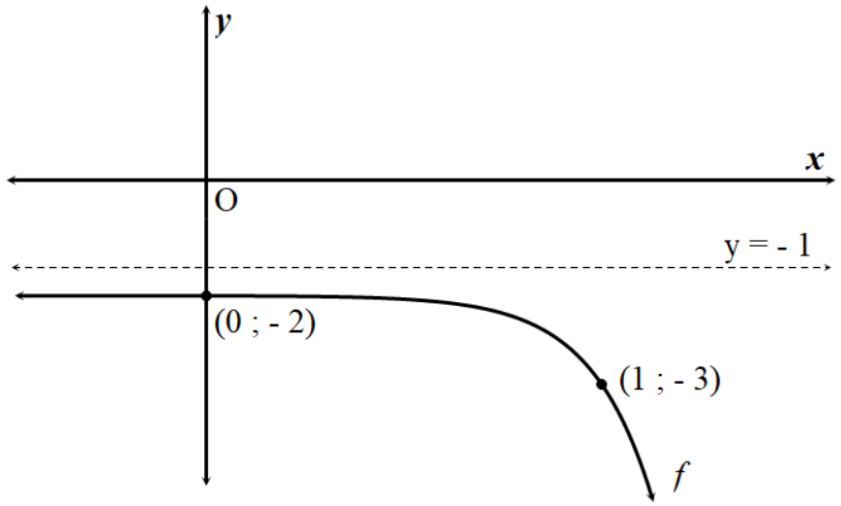
5.5 Calculate the value(s) of k for which $x^2-7x-8=k$ has no real roots. (2)

5.6 Determine the value(s) of x where $x.g(x)<0$. (1)

[14]

QUESTION 6

Sketched below is the function $f(x)=a.b^x+q.$ Points $(0;-2)$ and $(1;-3)$ are on the curve.



6.1 Calculate the values of a , b and $q.$ (5)

6.2 Determine the equation g if $f(x)$ is first reflected about the $x-$axis and then translated 1 unit down to generate $g(x)$. (2)

6.3 If $g(x)=2^x$, write down the equation of g^{-1} in the form $y=.....$ (2)

6.4 Sketch the graph g^{-1}, clearly indicating any intercepts with the axes and asymptotes that may exist.
Also indicate the point $\left(4;g^{-1}(4)\right)$ on your curve. (4)

6.5 Determine the value(s) of x for which $g^{-1}(x)\leq 2$. (2)

6.6 Write down the range of q if $q(x)=3f(x)$. (1)

[16]

QUESTION 7

7.1 Convert a nominal interest rate of 12% p.a. compounded monthly, to an effective rate of interest. (3)

7.2 You purchase a car for R100 000 and the depreciation rate will be 13% per annum compounded monthly on a reducing balance.

7.2.1 In which year will your car lose the greatest value? (1)

7.2.2 What will the value of your car be after 5 years? (3)

7.2.3 Calculate the cost of a similar new car be in 5 years' time, if inflation is expected to be 8% per annum for the next 5 years? (3)

7.3 Joe deposited R25 000 in a savings account. The interest rate given was 8% p.a. compounded quarterly for the first 3 years, and then 9% p.a. compounded monthly for the remaining time. 5 years after the initial deposit a withdrawal was made.

The balance at the end of 8 years is R37 000. How much was withdrawn? (6)

[16]

QUESTION 8

8.1 Differentiate $f(x)$ from first principles if $f(x)=1-2x^2$. (5)

8.2 Determine $\frac{dy}{dx}$ if:

8.2.1 $y=\frac{1}{2}x+\sqrt{x}-\frac{1}{x^2}$ (3)

8.2.2 $xy-y=x^2-1$ (4)

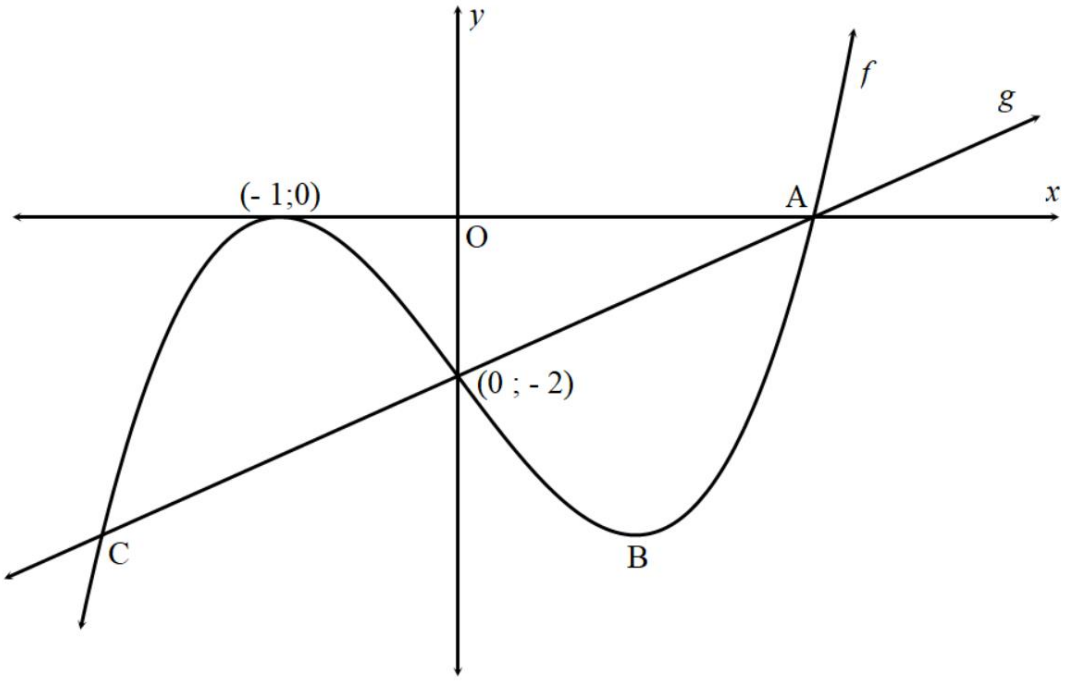
8.3 A function given by $f(x)=ax^2+\frac{b}{x}$ has a minimum value of 3 if $x=2$.

Calculate the values of a and b. (5)

[17]

QUESTION 9

9.1 The graph below represents the functions f and g with $f(x)=ax^3+cx-2$ and $g(x)=x-2$. A and $(-1;0)$ are the $x-$intercepts of $f(x)$.
The graphs of f and g intersect at A and C.



9.1.1 Determine the coordinates of A. (2)

9.1.2 Show, by calculation, that $a=1$ and $c=-3$. (4)

9.1.3 Determine the coordinates of B, a turning point of f. (3)

9.1.4 Write down the values of x for which $f'(x)<0$. (2)

9.2 The following information was given about the cubic function h:

- $h(-2)=h\left(\frac{1}{2}\right)=h(3)=0$;
- $h'(2)=h'\left(-\frac{4}{5}\right)=0$; and
- h increases for $x\in\left(-\frac{4}{5};2\right)$ only.

Draw a sketch graph of h, clearly indicating the information above. (4)

[15]

QUESTION 10

Bandile is designing a rocket which she shoots vertically into the air. The height it reaches is given by the formula $h(t) = 30t - 3t^2$, where height, h, is measured in metres and time, t, in seconds.

10.1 After how many seconds does it take for Veronica's rocket to reach its maximum height? (3)

10.2 Hence, determine what its maximum height the rocket reaches. (2)

10.3 Calculate the acceleration for this rocket. (2)

[7]

TOTAL: 150

HHS – GRADE **12**

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x\left[\left(1+i\right)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right) \qquad m = \tan\theta$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\left(x-a\right)^2 + \left(y-b\right)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$

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