HILLCREST HIGH SCHOOL

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INTERNAL ASSESSMENT

GRADE 12

MATHEMATICS
Paper 2
JUNE 2022

MARKS: 150 **TIME:** 3 hours

NAME:	TEACHER:

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	TOTAL
11	8	15	11	8	5	17	6	6	7	8	9	21	18	150
														MARKER

This booklet consists of 22 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 14 questions on 23 pages of an answer booklet. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera, which you have used in determining the answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

6. **This answer booklet is provided for answering the exam paper.** Use only this answer booklet. Write your name and teacher on the front of this booklet in the spaces provided. There is extra space after some question and at the end of the exam for any extra workings.

7. Number the answers correctly according to the numbering system used in this question paper. Clearly indicate if you are answering the question in the extra spaces provided.

8. It is in your own best interest to write legibly and to present your work neatly.

QUESTION 1

A survey was conducted at an all girls school in Durban.

1.1 The number of WhatsApp messages sent per day by a sample of 11 girls in the selected group was as follows:

25	33	40	42	45	50	53	57	61	70	85

1.1.1 Calculate the mean number of WhatsApp messaged sent per day by this sample of girls. (2)

1.1.2 Calculate the standard deviation for this sample. (2)

1.2 Using the same survey, details of the number of WhatsApp messages sent by a different sample of 39 girls are as follows:

- The lowest number of WhatsApp messages is 27 and the range is 60.
- The upper quartile is 72.
- The 10^{th} girl in the ranked data sent 35 messages and the 20^{th} girl sent 50 messages.

1.2.1 Determine the highest number of WhatsApp messages sent by this sample. (1)

1.2.2 How many girls sent more than 72 text messages? (1)

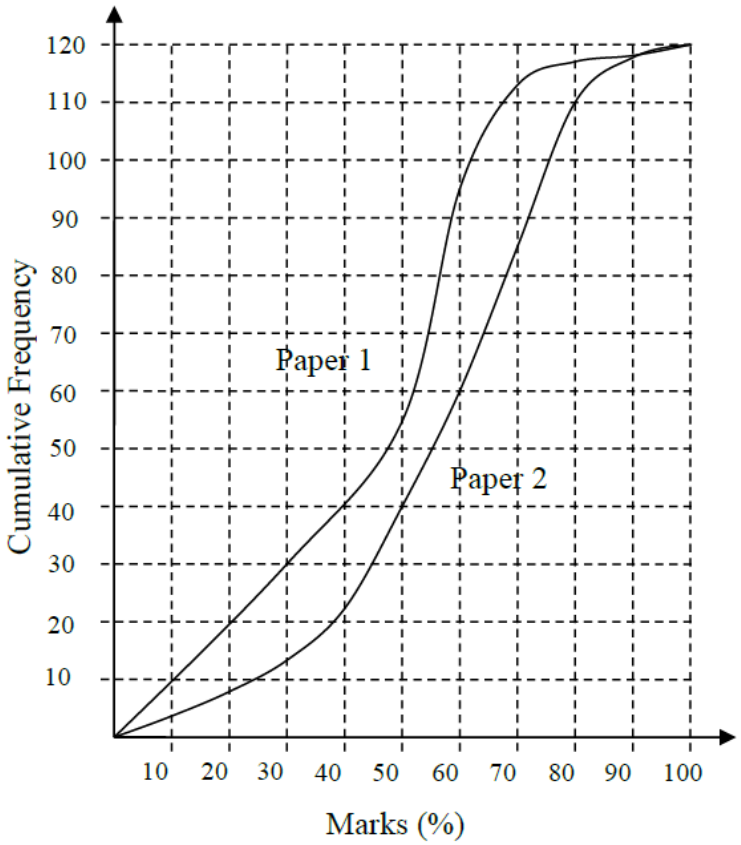
1.2.3 Use your information to draw a box-and-whisker plot of this data. Use a suitable scale and label all relevant points. (4)

1.2.4 Comment on the spread of your data, with reference to your plot in 1.2.3. (1)

[11]

QUESTION 2

The graph below compares the result of the two June Maths Exam papers for the Grade 12 group of 2018. The **same number of learners** wrote both papers.



2.1 How many learners wrote each exam? (1)

2.2 How many Grade 12s scored less than 40% for each paper? (2)

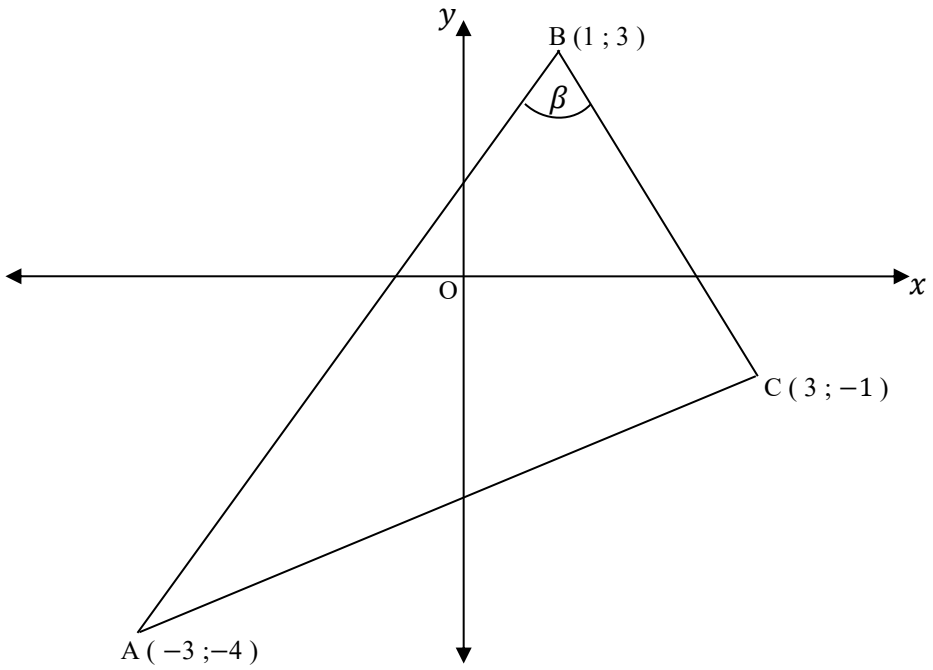
2.3 Determine the upper quartile for each paper. (2)

2.4 In which exam did the learners perform better? Justify your response by referring to your answers from 2.2 and 2.3. (3)

[8]

QUESTION 3

The diagram shows ΔABC with vertices $A(-3;-4)$, $B(1;3)$ and $C(3;-1)$. An equation for the line through B and C is $y = -2x + 5$.



3.1 Determine the midpoint of AC. (2)

3.2 Calculate the magnitude of β, as shown in the diagram. (5)

3.3 Calculate the co-ordinates of D, the x - intercept of the straight line through A and C. (4)

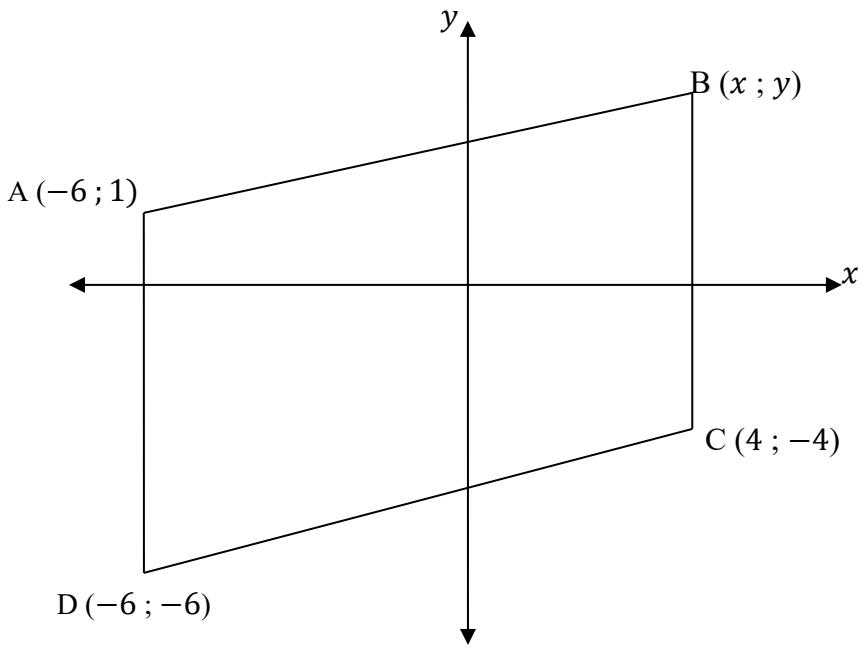
3.4 Is BC perpendicular to AC? Justify your answer fully. (2)

3.5 Write down the equation of the perpendicular bisector of AC. (2)

[15]

QUESTION 4

In the diagram below, quadrilateral ABCD is a parallelogram.



4.1 Write down the length of AD. (1)

4.2 Write down the co-ordinates of B. (2)

4.3 An equation of the line passing through A and C is $2y = -x + p$

4.3.1 Determine the value of p. (2)

4.3.2 Write down the co-ordinates of the y - intercept of the line AC. (2)

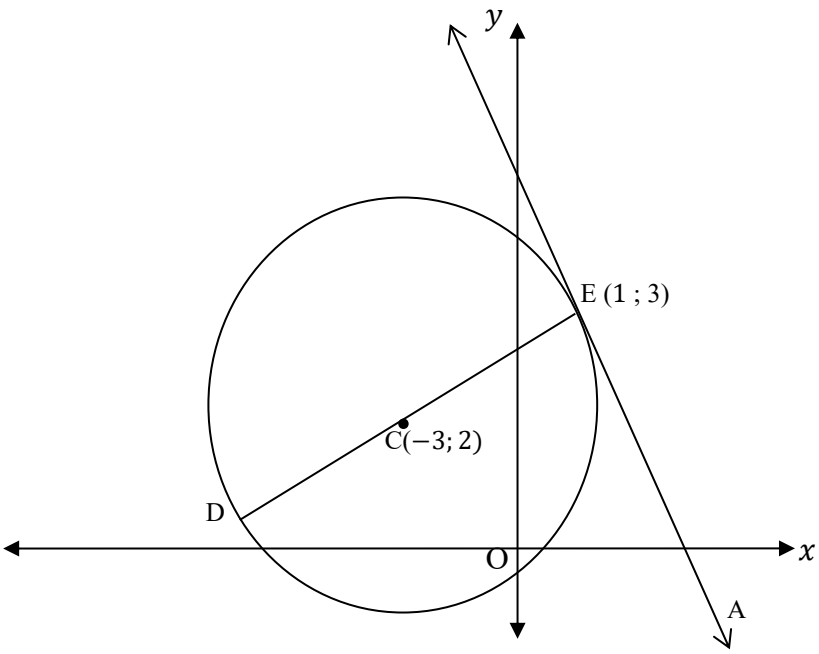
4.4 Is ABCD a rhombus?
Show calculations and give reasons to justify your answer. (4)

[11]

EXTRA SPACE

QUESTION 5

In the figure below, the centre C$(-3\ ;\ 2)$ of the circle lies on the diameter DCE.
AE is a tangent to the circle at E (1 ; 3).



5.1 Calculate the length of the radius of the circle. Leave the answer in surd form if necessary. (2)

5.2 Write down the coordinates of D. (2)

5.3 If F$(a\ ;\ b)$ is a point that is equidistant from C and E, show that $b = -4a - \frac{3}{2}$. (4)

[8]

QUESTION 6

A $(-2\ ;\ 3)$ and B $(4\ ;-1)$ are the end points of a chord of a circle. An equation of the tangent to the circle at A is $3x - 2y + 4 = 0$.

Giving reasons where necessary, prove that AB is a diameter of the circle.

[5]

QUESTION 7

7.1 Simplify the following:

$$\frac{\cos(180°+x).\tan x}{\cos(90°-x)} \quad (4)$$

7.2 If $\sin 20° = t$, express the following in terms of t:

7.2.1 $\sin 200°$ (2)

7.2.2 $\cos 70°$ (2)

7.2.3 $2\cos^2 10°.\tan 10°$ (3)

7.3 Given $\cos\beta = \frac{12}{13}$ with $\beta \epsilon [180°; 360°]$ and $\alpha = 45°$.
Determine the value of $\cos(\alpha + \beta)$ **without the use of a calculator.** (6)

[17]

QUESTION 8

Given that $\tan^2 P = \frac{1-\cos 2P}{1+\cos 2P}$.

8.1 Prove the identity. (4)

8.2 For what value(s) of P is the given identity undefined? (2)

[6]

QUESTION 9

Given that $2\sin x.\cos x + 2\sin x + \cos^2 x + \cos x = 0$.

9.1 Show that the above equation can be written as:

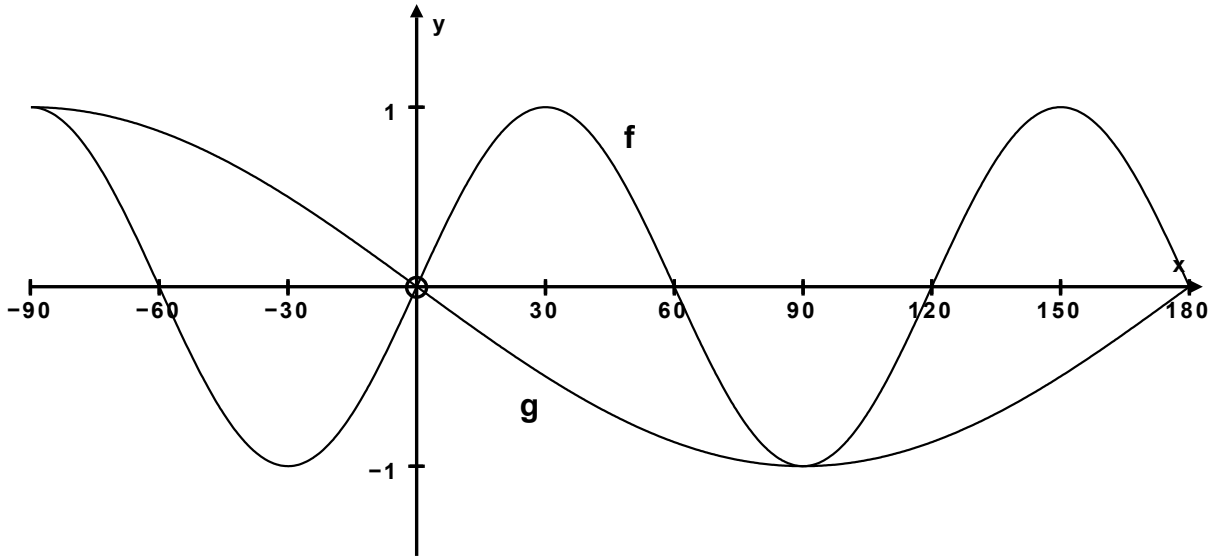
$(\cos x + 1)(2\sin x + \cos x) = 0$ (1)

9.2 Hence, determine the general solution for this equation. (5)

[6]

QUESTION 10

Study the graph of $f(x) = a \sin kx$ and $g(x) = \cos(x + p)$ for the domain $x \in [-90° ; 180°]$ that are drawn on the same set of axes below:



10.1 Determine the values of a, k and p. (3)

10.2 g can also be written as a sine function. Give that equation in the form $y = \ldots$ (1)

10.3 If g is translated $60°$ left to form h, give an equation of h. (1)

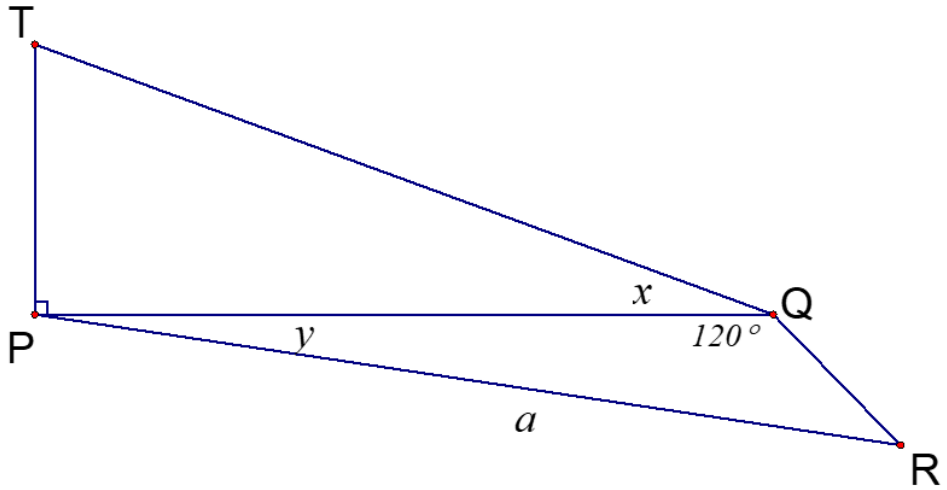
10.4 Using the graph above, for which value(s) of x in the above domain is $f(x) > g(x)$. (2)

[7]

QUESTION 11

In the sketch, TP represents the height of the building. The foot of the building P and the points Q and R are in the same horizontal plane. From Q, the angle of elevation to the top of the building is x.

$P\hat{Q}R = 120°$, $Q\hat{P}R = y$ and the distance between P and R is a metres.



11.1 Determine $\hat{R}$ in terms of y. (1)

11.2 Determine the length of PQ in terms of a and y. (2)

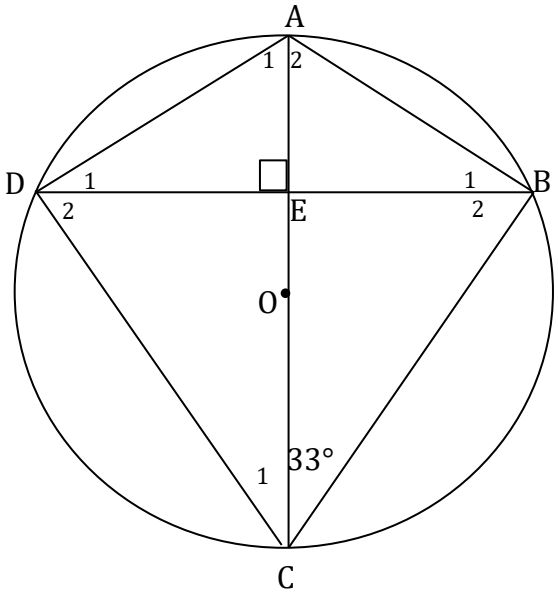
11.3 Hence, or otherwise, prove that the length of TP is

$$TP = a\tan x\,(\cos y - \frac{\sqrt{3}}{3}\sin y)$$ (5)

[8]

QUESTION 12

In the diagram below, AC is the diameter of the circle with centre O. AC and chord BC intersect at E. Points A, B, C and D are on the circumference on the circle.



12.1 If $A\hat{C}B = 33°$, determine, with reasons, the size of:

12.1.1 $\hat{A}_1$ (3)

12.1.2 $\widehat{D}_2$ (2)

12.2 If the diameter AC is 20 cm and DB is 16 cm, determine, with reasons, the length of OE. (4)

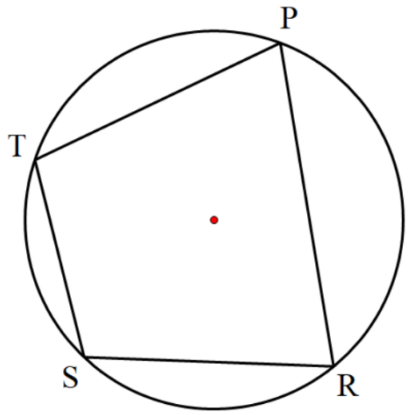
[9]

QUESTION 13

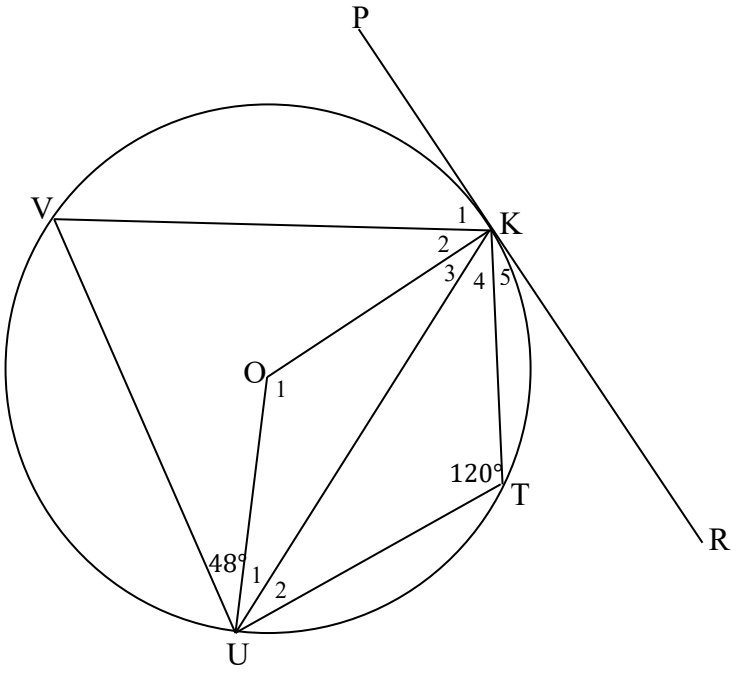
13.1 The diagram below shows circle centre O with cyclic quadrilateral PRST.

Prove the theorem which states: $\hat{TPR} + \hat{TSR} = 180°$.

(5)



13.2 In the diagram below, O is the centre of the circle passing through K, T, U and V. PKR is a tangent to the circle at K. $\hat{OUV} = 48°$ and $\hat{KTU} = 120°$.



The table below contains 7 statements concerning the sketch.

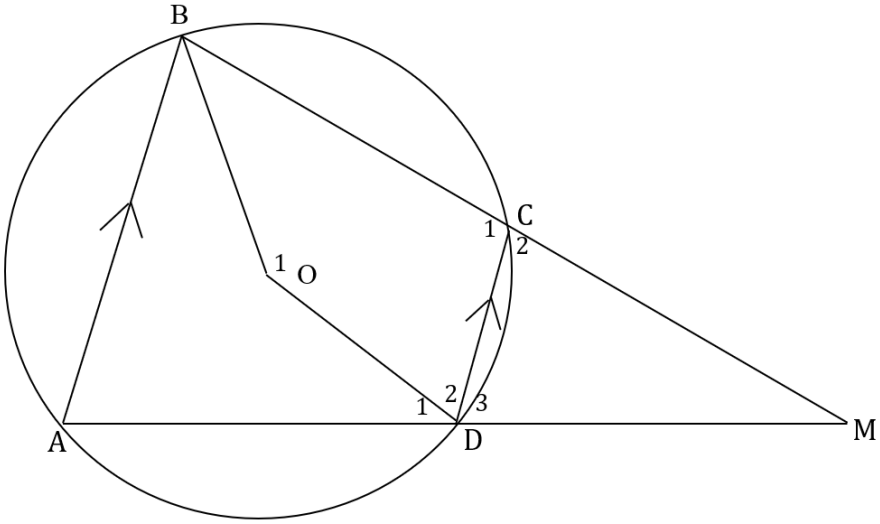
Complete the table by giving reasons for these statements. (5)

Statement	**Reason**
$\hat{V} = 60°$	13.2.1
$\hat{O}_1 = 120°$	13.2.2
OU = OK	Radii
$\hat{U}_1 = \hat{K}_3$	13.2.3
$\hat{U}_1 = 30°$	Sum of angles in a Δ
$\hat{K}_1 = 78°$	13.2.4
$\hat{K}_2 = 12°$	13.2.5

13.3 Complete the following statement:

" The exterior angle of a cyclic quadrilateral is . . . . . " (1)

13.4 In the figure below ABCD is a cyclic quadrilateral with AB || DC.
O is the centre of the circle and BC and AD are produced to meet at M



13.4.1 If $\hat{O}_1 = 2x$, write down, with reasons, TWO angles that are equal to x. (4)

13.4.2 Prove that MC = MD. Give reason(s) for statement(s). (3)

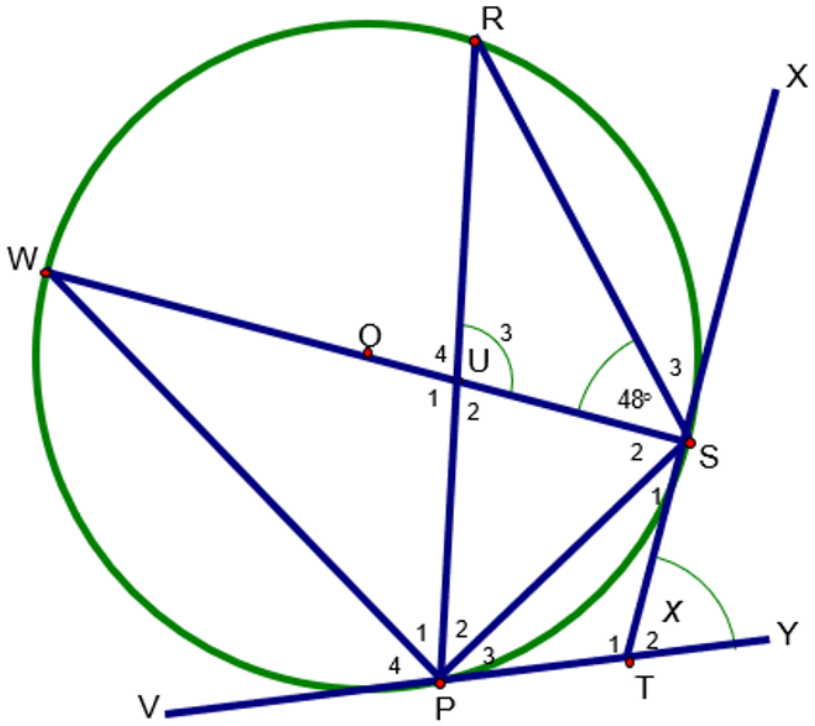
13.4.3 Express $\hat{M}$ in terms of x, giving a reason for your answer. (1)

13.4.4 Prove, with reasons, that BODM is a cyclic quadrilateral. (2)

[21]

QUESTION 14

14.1 In the diagram below, VY and TX are tangents to circle centre O at P and S respectively. $\hat{T}_2 = x$ and WOS is the diameter of the circle. Points P, R, S and W are on the circumference of the circle.



Answer the questions that follow, **giving reasons** for each of your statements:

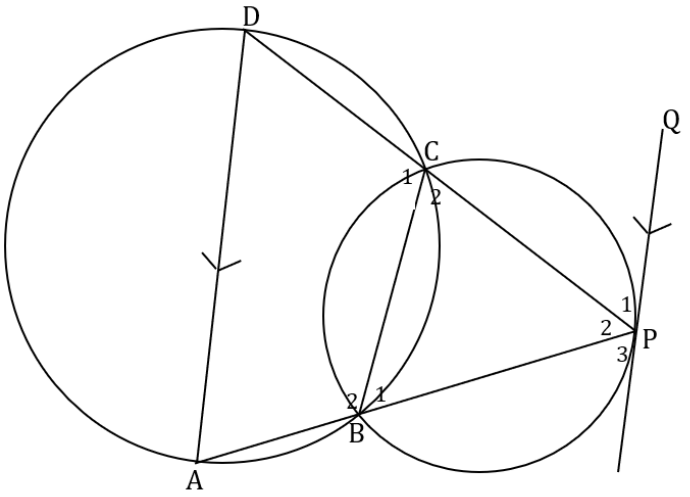
14.1.1 Determine the size of $\hat{P}_3$ in terms of x. (4)

14.1.2 Write down two other angles each equal to $\hat{P}_3$. (3)

14.1.3 Calculate the size of $\hat{S}_2$ in terms of x. (2)

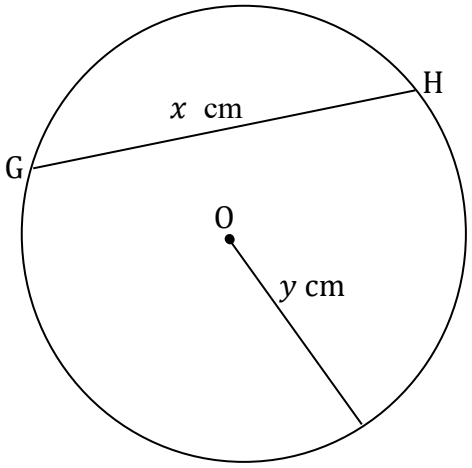
14.1.4 Is PTSU a cyclic quadrilateral or not? Explain your answer fully. (3)

14.2 AB and DC are chords of cyclic quad ABCD are produced to meet at P. It is given that PQ || AD.



Prove, with reasons, that PQ is a tangent to circle PBC. (4)

14.3 O is the centre of the circle alongside.



State, in terms of x and/or y:

14.3.1 The maximum length of chord GH. (1)

14.3.2 The relationship between x and y for which it would become impossible to draw a circle with these dimensions. (1)

[18]

TOTAL [150]

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x\left[\left(1+i\right)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right) \qquad m = \tan\theta$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\left(x-a\right)^2 + \left(y-b\right)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$