Gr 12

HILLCREST HIGH SCHOOL

MATHEMATICS

PAPER 2

MAY / JUNE 2011

MARKS: 100

TIME: 2 Hours

This question paper consists of 7 pages, an information sheet and a diagram sheet.

INSTRUCTIONS AND INFORMATION

1. This question paper consists of 7 questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.

3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.

5. Diagrams are NOT necessarily drawn to scale.

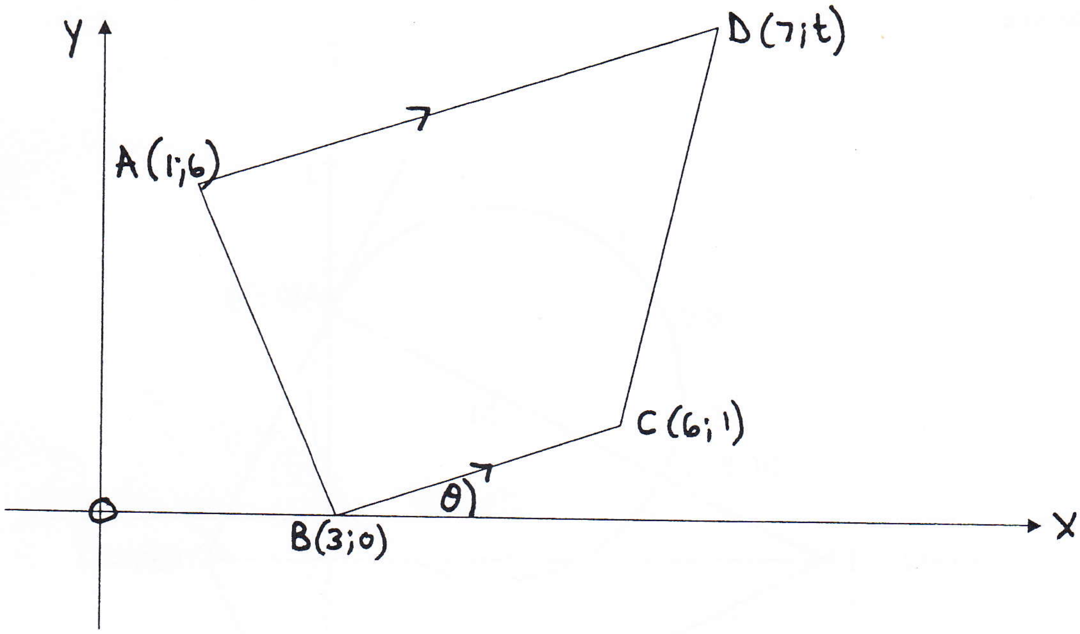
6. ONE diagram sheet for answering QUESTION 3.2.2, QUESTION 7.1, is included at the end of this question paper. Write your name on this sheet in the space provided and hand them in together with your ANSWER BOOK.

7. Number the answers correctly according to the numbering system used in this question paper.

8. It is in your own interest to write legibly and to present the work neatly.

QUESTION 1

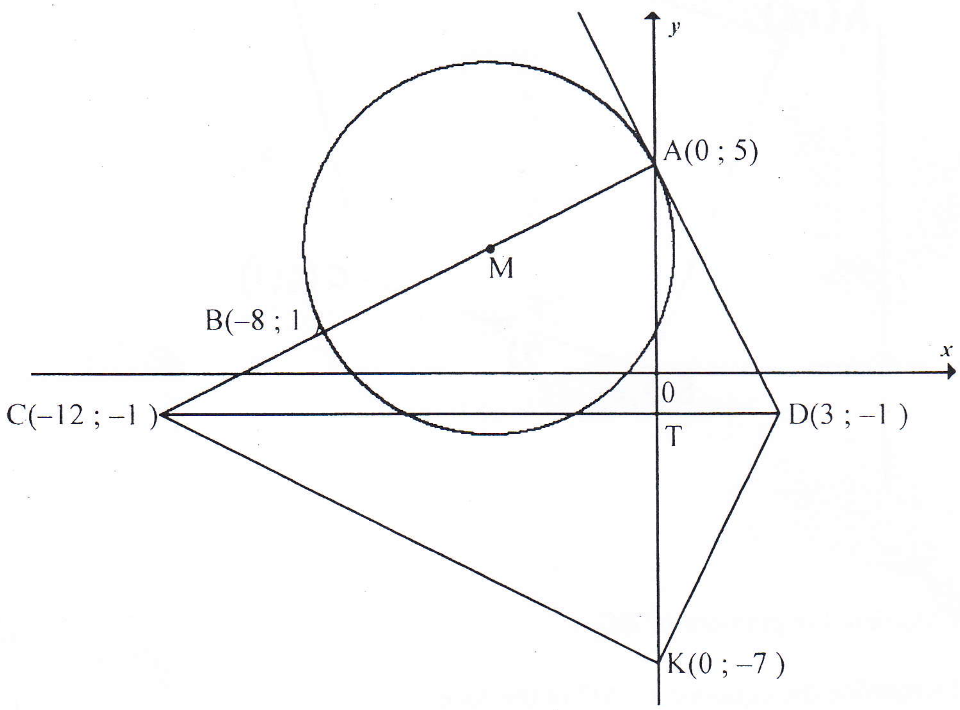
ABCD is a quadrilateral with vertices A(1;6), B(3;0), C(6;1) and D(7; t) in a Cartesian plane. AD || BC.



1.1	Calculate the gradient of BC.	(2)
1.2	Determine the equation of AD in the form y=....	(3)
1.3	Show that t=8.	(2)
1.4	Show that BA is perpendicular to BC.	(3)
1.5	Determine θ, the angle of inclination of BC.	(3)
		[13]

QUESTION 2

A(0 ; 5) and B(–8 ; 1) are two points on the circumference of the circle centre M, in a Cartesian plane. M lies on AB. DA is a tangent to the circle at A. The coordinates of D are (3 ; –1) and the coordinates of C are (–12 ; –1). Points C and D are joined. K is the point (0 ; –7). CTD is a straight line.



2.1 Show that the coordinates of M, the midpoint of AB, are (–4 ; 3). (1)

2.2 Determine the equation of the tangent AD. (4)

2.3 Determine the length of AM. (3)

2.4 Determine the equation of circle centre M in the form $ax^2 + by^2 + cx + dy + e = 0$ (4)

2.5 Quadrilateral ACKD is one of the following:

parallelogram; kite; rhombus; rectangle

Which one is it? Justify your answer. (4)

[16]

QUESTION 3

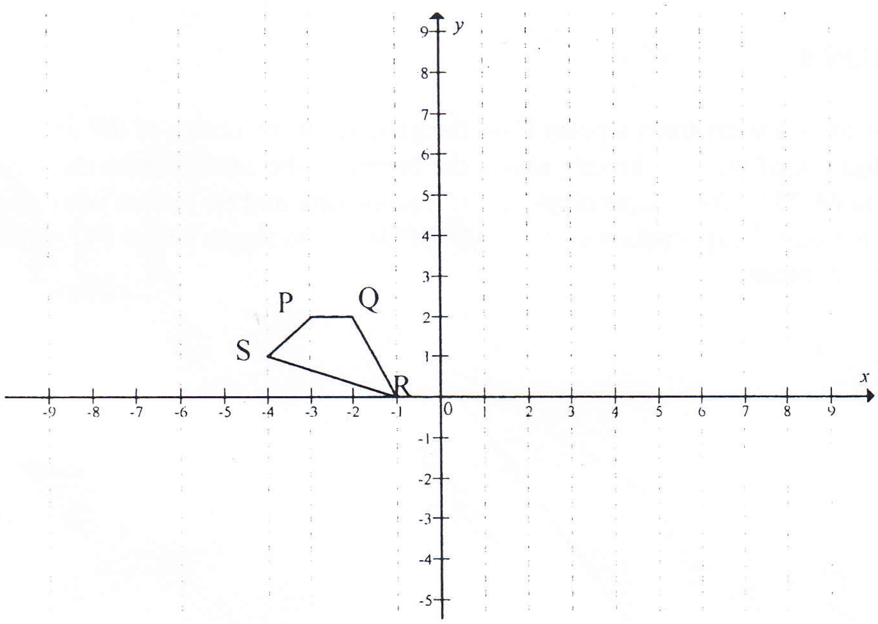
The point $P(\sqrt{3}\ ; -2)$ lies in a Cartesian plane.

3.1 Determine the coordinates of the image of P if:

3.1.1 P is reflected about the y-axis (2)

3.1.2 P is rotated about the origin through 180° in an anticlockwise direction (2)

3.2 The vertices of a polygon PQRS are shown in the grid below. The coordinates are P(–3 ; 2), S(–4 ; 1), R(–1 ; 0) and Q(– 2 ; 2). Each of the points of PQRS in the grid below is rotated about the origin in a clockwise direction through an angle 90°.



3.2.1 Write down the coordinates of Q', the image of Q. (2)

3.2.2 Sketch and label the vertices of the image $P'\ Q'\ R'\ S'$ of PQRS on the grid provided on DIAGRAM SHEET 1. (4)

3.2.3 The polygon $P'Q'R'S'$ is then enlarged through the origin by a scale factor of 2 to give the polygon $P''Q''R''S''$. Write down the coordinates of P'' the image of P'. (2)

3.2.4 State whether the transformation from PQRS to $P''Q''R''S''$ is rigid or not. Give a reason for your answer. (2)

3.2.5 Write down the general transformation of a point $(x\ ;\ y)$ in PQRS to $(x''\ ;\ y'')$ after PQRS has undergone the above two transformations, namely rotation through 90° clockwise followed by an enlargement through the origin by a factor of 2. (3)

3.2.6 Calculate the ratio of area PQRS : Area $P''Q''R''S''$. (2)

[19]

QUESTION 4

Simplify each of the following to a single trigonometric ratio: (Show ALL the calculations)

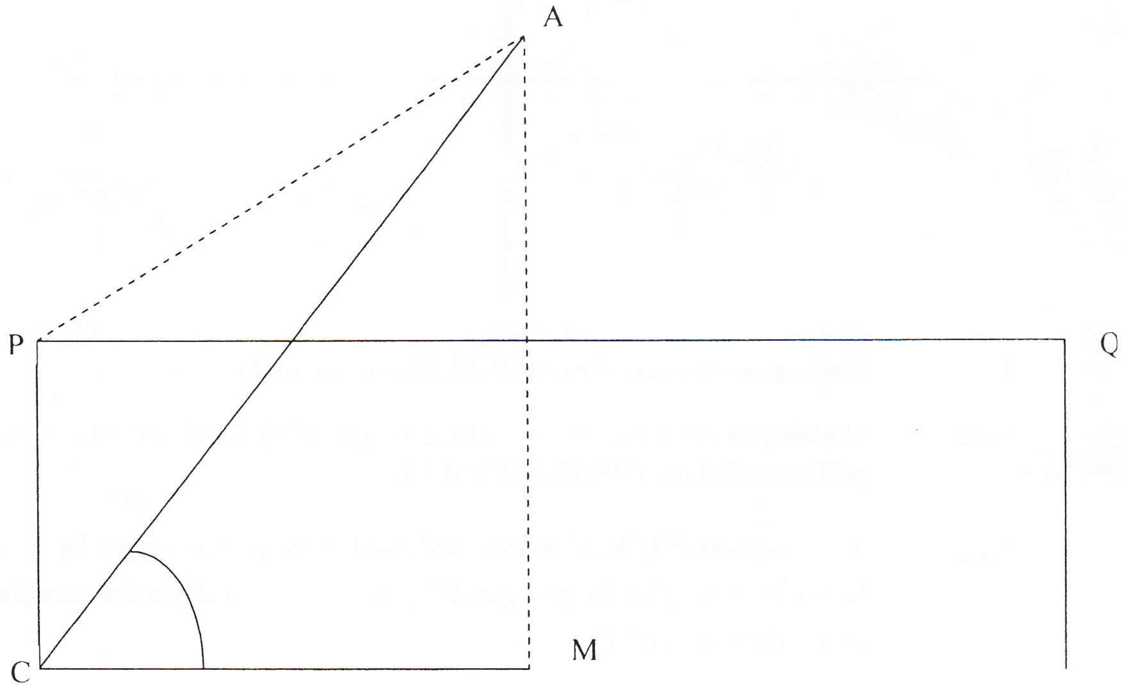
4.1 $$\frac{\tan(180^\circ + x)Cos(360^\circ - x)}{\sin(180^\circ - x)\cos(90^\circ + x) + Cos(540^\circ + x)Cos(-x)}$$ (8)

4.2 $$\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$$ (5)

[13]

QUESTION 5

The angle of elevation from a point C on the ground, at the centre of the goalpost, to the highest point A of the arc, directly above the centre of the Moses Mabhida soccer stadium, is $64{,}75^\circ$. The soccer pitch is 100 metres long and 64 metres wide as prescribed by FIFA for world cup stadiums. Also AC $\perp$ PC. In the figure below PQ = 100metres and PC = 32 metres.



5.1 Determine AC (3)

5.2 Determine $P\hat{A}C$ (3)

[6]

QUESTION 6

6.1 If $\sin 23° = p$, write down the following in terms of p. Do NOT use a calculator.

6.1.1 $\cos 113°$ (2)

6.1.2 $\cos 23°$ (2)

6.1.3 $\sin 46°$ (2)

6.2 It is known that $13\sin\alpha - 5 = 0$ and $\tan\beta = -\frac{3}{4}$ where $\alpha \in [90°; 270°]$ and $\beta \in [90°; 270°]$. Determine, without using a calculator, the values of the following:

6.2.1 $\cos\alpha$ (3)

6.2.2 $\cos(\alpha + \beta)$ (5)

6.3 Solve for $x \in [0°; 360°]$ if $\frac{1}{2}\cos x = 0{,}435$. (3)

[17]

QUESTION 7

Consider the functions defined by $f(x) = \sin 2x$ and $g(x) = \frac{1}{2}\tan x$ for $x \in [-90°;180°]$.

7.1 The graphs have been sketched on the Diagram Sheet. Label all the points on the graph and label which graph is $f(x)$ and which is $g(x)$. (4)

7.2 Calculate the x-coordinates of the points of intersection of f and g. (8)

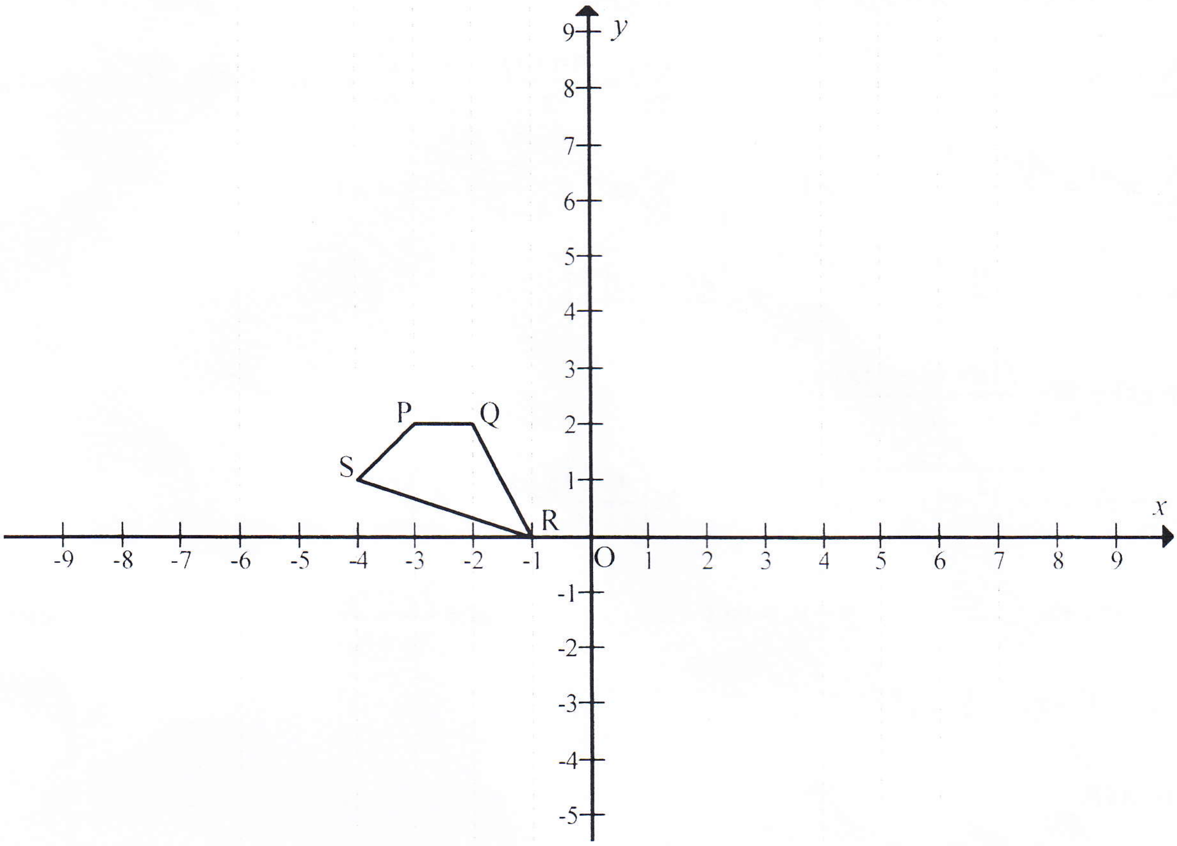
7.3 Determine the values of x for which $g(x) > f(x)$. (3)

[15]

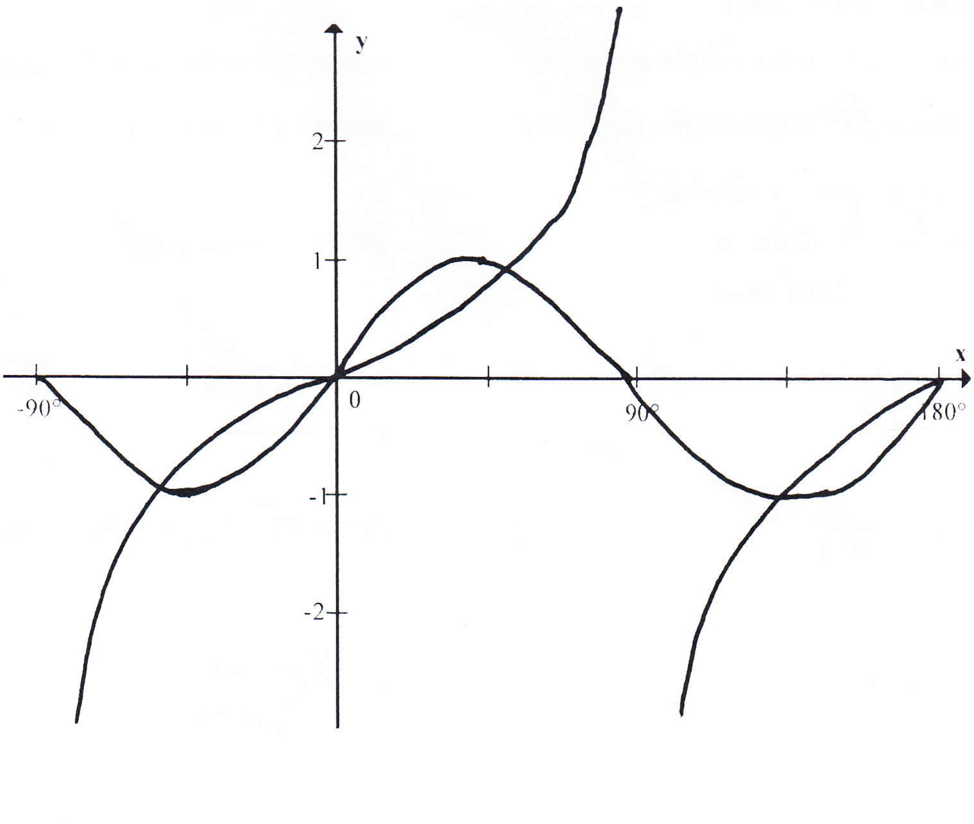
NAME :

DIAGRAM SHEET 1

QUESTION 3.2.2



QUESTION 8.1



INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad \sum_{i=1}^{n} (a + (i-1)d) = \frac{n}{2}(2a + (n-1)d)$

$\sum_{i=1}^{n} ar^{i-1} = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad \sum_{i=1}^{\infty} ar^{i-1} = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$