HILLCREST HIGH SCHOOL

MATHEMATICS

PAPER 1

Gr 12

JUNE 2013

MARKS: 120

TIME: $2\frac{1}{2}$ Hours

This question paper consists of 5 pages, and an information sheet.

QUESTION 1

Solve for x :

1.1 $3x^2 = 2(x + 5)$, giving your answer correct to one decimal digit. (4)

1.2 $125^{3x-2} = (5^2)^{4x+10}$ (4)

[8]

QUESTION 2

2.1 Given $f(x) = x^2 + 5x$. Determine $f'(x)$ from first principles. (5)

2.2 Determine:-

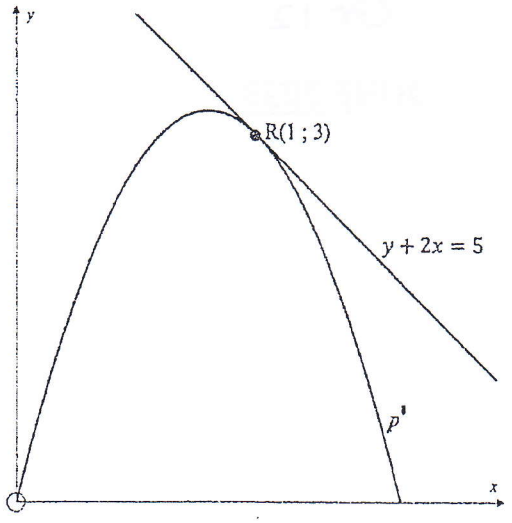
2.2.1 $\frac{dy}{dx}$ if $y = \left(\frac{x}{3} + \frac{3}{x}\right)^2$ (4)

2.2.2 $\frac{d}{dx}\left[\frac{\sqrt{x^3}-5x+2}{\sqrt{x}}\right]$ (5)

[14]

QUESTION 3

In the figure below, $y + 2x = 5$ is a tangent to $p'(x) = ax^2 + bx, a \neq 0$ at the point R(1 ; 3).



3.1 Calculate $p''(x)$, in terms of a and b. (1)

3.2.1 What is the gradient of p' at point R? (when $x = 1$) (1)

3.2.2 Hence, show that the value of a and b are as follows $a = -5$ and $b = 8$. (6)

3.3 Find the x co-ordinate of the turning points of p. (3)

[11]

QUESTION 4

GIVEN: $g(x) = x^3 - x^2 - 8x + 12$

4.1 Calculate the intercepts of the graph of g with the axes. (5)

4.2 Determine the co-ordinates of the turning points of g. (5)

4.3 Sketch the graph of g. Clearly indicating the intercepts with the axes and the turning points. (4)

4.4 Determine the value(s) of x for which:-

4.4.1 $g(x) \geqslant 0$ (2)

4.4.2 $g'(x) \leq 0$ (3)

4.5 If $h(x) = g(-x)$, then write down the equation of h. (2)

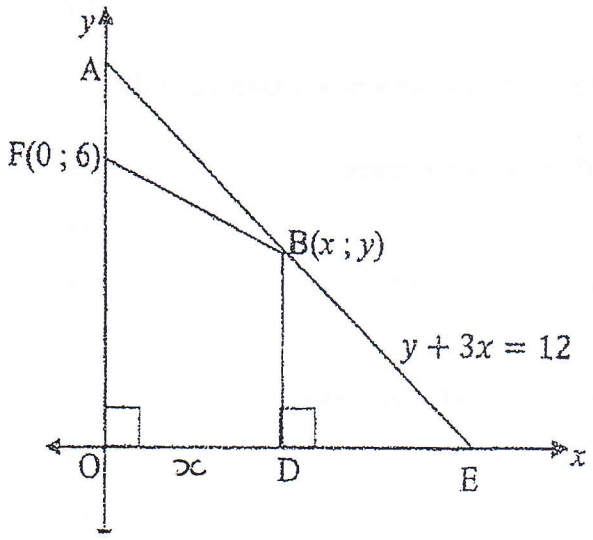
4.6 Given that one of the roots of $g(x) = k$ is equal to 0, find the value of k. (1)

[22]

QUESTION 5

In the diagram below, AE is defined by $y + 3x = 12$ and $x, y \geq 0$.

B$(x\,;y)$ is any point on AE. F is the point (0 ; 6). BD is parallel to the y-axis.



5.1 Write down the y-coordinate of B in terms of x. (1)

5.2 Given: Area of a trapezium $= \frac{1}{2}(sum\ of\ parallel\ sides)h$

Show that the area of quadrilateral BDOF is given by $A = -\frac{3}{2}x^2 + 9x$. (3)

5.3 Calculate the maximum area of quadrilateral BDOF. (4)

[8]

QUESTION 6

Consider the function $f(x) = \frac{4}{x+1} - 1$

6.1 Calculate the intercepts of the graph f with the axes. (4)

6.2 Sketch the graph of f on DIAGRAM SHEET 1. (3)

6.3 Write down the domain of f. (2)

6.4 Describe the transformation of f to g if $g(x) = \frac{-4}{x+1} + 1$ (2)

[11]

QUESTION 7

7.1 Given the sequence 4; x; 32; ...

Determine the value of x if the sequence is

7.1.1 Arithmetic (2)

7.1.2 Geometric (3)

7.2 The following arithmetic sequence is given: 20; 23; 26; 29; ...101

7.2.1 How many terms are there in this sequence? (3)

The even numbers are removed from the sequence to make a new sequence of the remaining terms.

7.2.2 Determine the common difference of this new sequence. (1)

7.2.3 Determine n in this new sequence. (1)

7.2.4 Calculate the sum of the terms of this new sequence. (3)

7.3 $3x + 1$; $2x$; $3x - 7$ are the first three terms of an arithmetic sequence.

Calculate the value of x. (2)

[15]

QUESTION 8:

8.1 Consider the geometric sequence: 4; -2; 1; ...

8.1.1 Determine the next term of the sequence. (1)

8.1.2 Determine the general term of the above sequence. (2)

8.1.3 What will the value of the 15^{th} term be? (2)

8.2 Calculate the sum to infinity of the series $4 - 2 + 1\ldots$ (3)
[8]

QUESTION 9

Determine the value of n such that $\sum_{k=1}^{n}(3 \div 2k) = 896.$

[6]

QUESTION 10

For which values of a is the infinite series $2(3a\text{-}1) + 2(3a - 1)^2 + \ldots$ convergent [3]

QUESTION 11

An instant coffee manufacturer produces two types of instant coffee: INSTANTO and BLITZ. There are two processes in the manufacture of instant coffee, roasting and granulating. Each 100kg of INSTANTO requires 2 hours roasting and 3 hours granulating while BLITZ needs 4 hours roasting and 2 hours in the granulating machines. The roasting and granulating machines can each be run up to a maximum of 8 hours per day.

The profit on INSTANTO is R100 per 100kg while BLITZ realizes a profit of R150 per 100kg.

Let a 100kg of INSTANTO coffee be x and 100kg of BLITZ coffee be y.

11.1 List the FOUR inequalities (including the implicit constraints) that result from the above constraints. (4)

11.2 Graph the inequalities on the DIAGRAM SHEET attached to your answer booklet and clearly indicate the feasible region. (5)

11.3 Write down an expression for the total profit per day. (1)

11.4 With the use of a search line, state the amounts of INSTANTO and BLITZ the manufacturer should produce per day in order to maximize profits. (4)
[14]

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x\,;\,y) \to (x\cos\theta + y\sin\theta\,;\, y\cos\theta - x\sin\theta) \qquad (x\,;\,y) \to (x\cos\theta - y\sin\theta\,;\, y\cos\theta + x\sin\theta)$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$