HILLCREST HIGH SCHOOL

MATHEMATICS

Gr 12 **PAPER 2**

MAY / JUNE 2013

MARKS: 120

TIME: 2½ Hours

This question paper consists of 7 pages and an information sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 10 questions. Answer ALL the questions.
2. Show ALL calculations, diagrams, graphs etc., clearly, which you have used in determining the answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Number the answers correctly according to the numbering system used in this question paper.
6. Diagrams are NOT necessarily drawn to scale.
7. Write neatly and legibly.
8. An information sheet, with formulae, is included at the end of the question paper.

QUESTION 1

Given the points: A (-1;0) B (0;3) C (8;11) D $(x;y)$

Determine:

1.1 the length of AC (2)

1.2 the gradient of BC (2)

1.3 M, the midpoint of AC (2)

1.4 D $(x;y)$ if ABCD is a parallelogram (4)

1.5 Prove that the points B, C and E (-2;1) are collinear (5)

[15]

QUESTION 2

Given the points A (1;3); B (3;2) and C (-1;-1), determine the following:

2.1 the equation of the line AB (3)

2.2 the equation of the line passing through C, parallel to AB (2)

2.3 the equation of the line passing through B, perpendicular to AB (3)

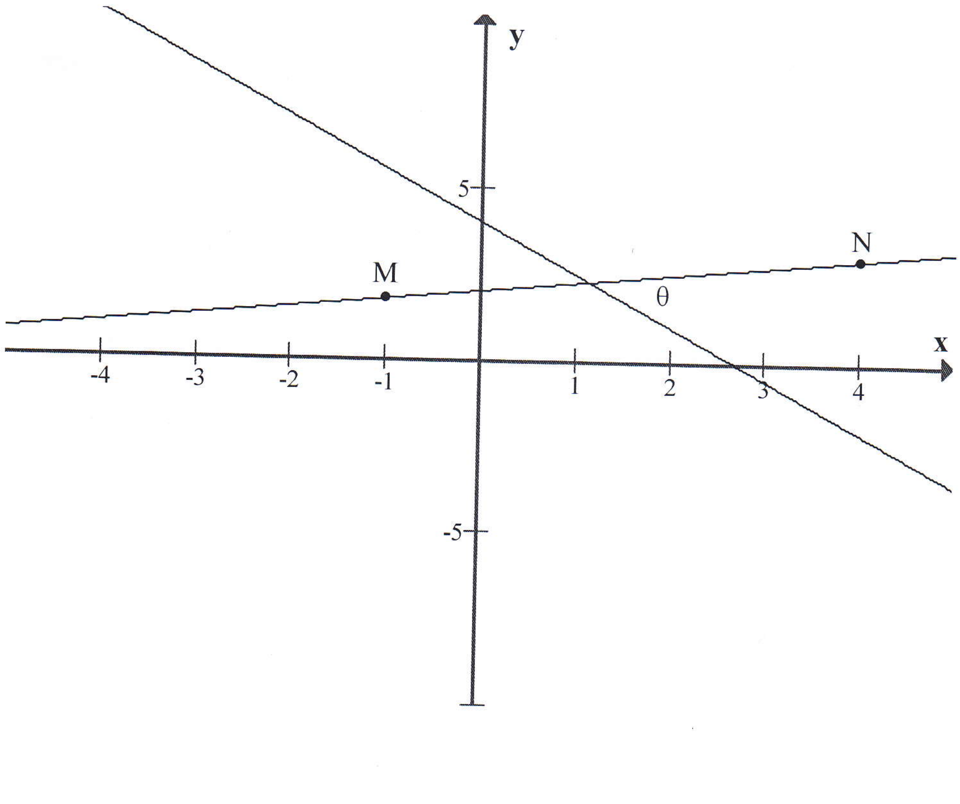
2.4 the co-ordinates for A, B and C if the points are reflected about the x-axis (3)

[11]

QUESTION 3

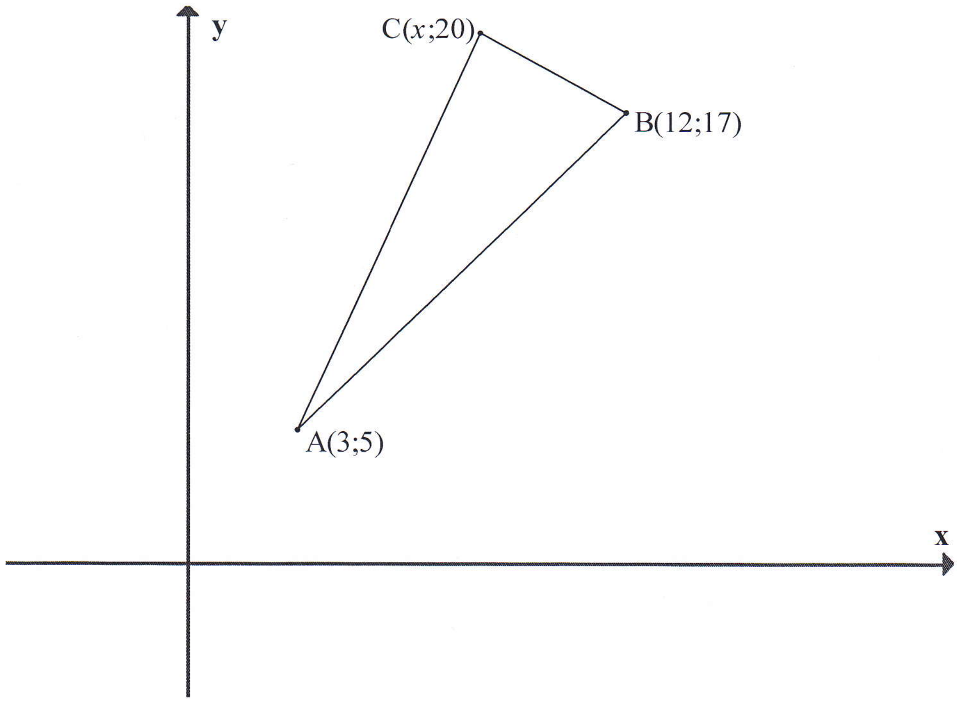
Determine the acute angle (θ), correct to 1 decimal place) between the line passing through the points M (-1; $1\frac{3}{4}$) and N (4;3); and the straight line $y = -\frac{3}{2}x + 4$.

[7]



QUESTION 4

In the diagram below, ΔABC is a right-angled triangle with CB $\perp$ AB. ΔABC has vertices A(3 ; 5), B(12; 17) and C(x; 20) on the Cartesian plane.



4.1 Determine the value of x. (6)

4.2 If BC = 5 units, calculate the perimeter of ΔABC. (Leave the answer is simplest surd form.) (6)

[12]

QUESTION 5

5.1 Given: $\cos\alpha = -\frac{3}{5}$ where $0^\circ \leq \alpha \leq 180^\circ$

With the aid of a sketch and without the use of a calculator, calculate:

5.1.1 $\tan\alpha$ (3)

5.1.2 $\cos(90^\circ + \alpha)$ (2)

5.1.3 $\sin 2\alpha$ (3)

5.1.4 Using a calculator, determine the size of α, correct to 1 decimal place. (2)

5.2 If $\sin 54^\circ = p$, then express the following in terms of p:

5.2.1 $\sin 594^\circ$ (2)

5.2.2 $\sin 108^\circ$ (4)

5.2.3 $\cos 18^\circ$ (4)

[20]

QUESTION 6

6.1 Simplify each of the following to a single trigonometric ratio: (Show ALL the calculations)

6.1.1 $$\frac{\tan(180^\circ + x)\cos(360^\circ - x)}{\sin(180^\circ - x)\cos(90^\circ + x) + \cos(540^\circ + x)\cos(-x)}$$ (7)

6.1.2 $$\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x}$$ (6)

6.2 Simplify fully:
$(\cos 15^\circ - \sin 15^\circ)(\cos 15^\circ + \sin 15^\circ)$ (3)

[16]

QUESTION 7

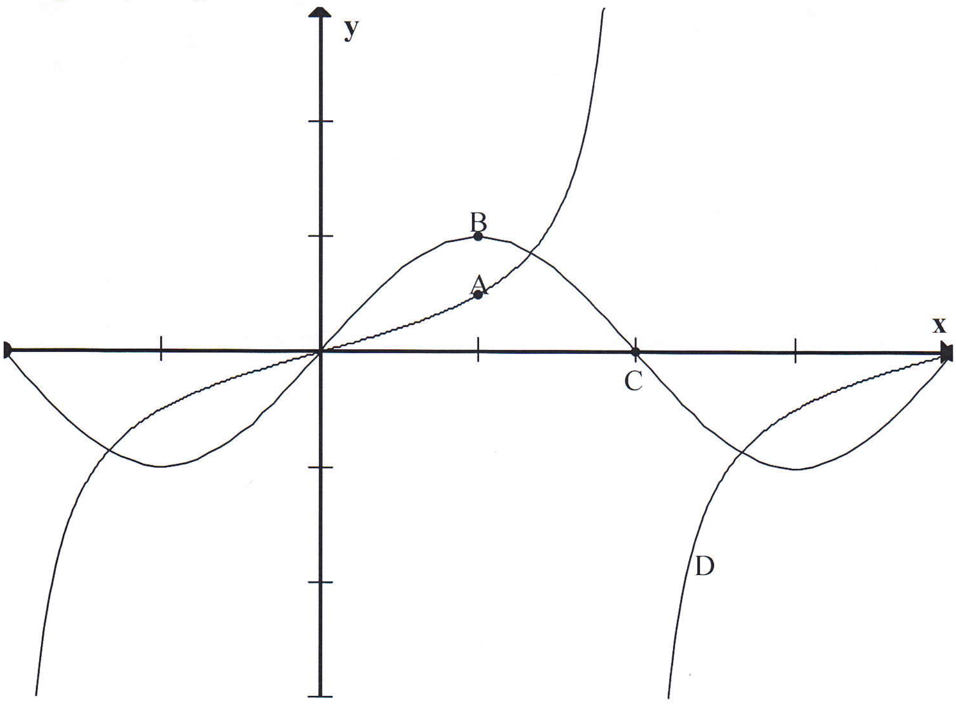
7.1 Prove the identity: $\frac{\sin 2x}{1 + \cos 2x} = \tan x$ (5)

7.2 Determine the value(s) of x for which the identity in Question 6.1 will be undefined? (3)

[8]

QUESTION 8

Consider the functions defined by $f(x) = \sin 2x$ and $g(x) = \frac{1}{2}\tan x$ for $x \in [-90^\circ; 180^\circ]$.



8.1 The graphs have been sketched above. Write down the co-ordinates for A, B and C. (3)

8.2 Name the graph represented by D. (1)

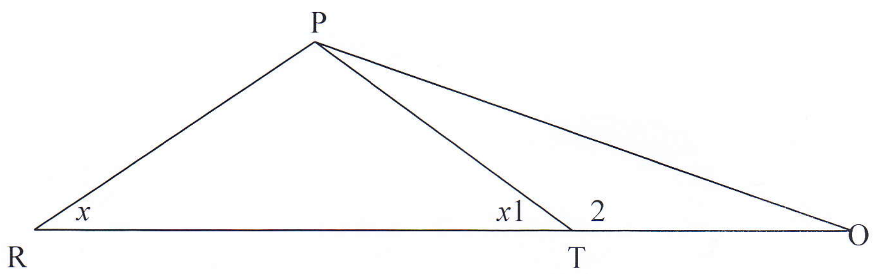
8.3 Calculate the x-coordinates of the points of intersection of f and g. (8)

8.4 Determine the values of x for which $g(x) > f(x)$. (3)

[15]

QUESTION 9

In the figure alongside, ΔPRT has PR = PT.



If TO = ½RT = a and $\hat{R} = \hat{T}_1 = x$,

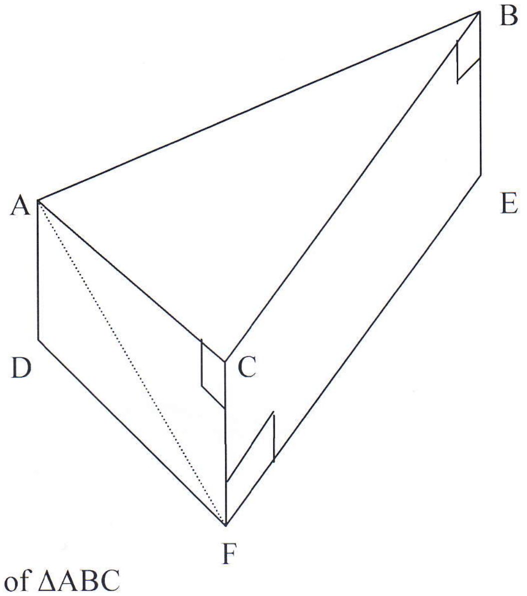
9.1 Prove that PT = $\dfrac{a}{\cos x}$ (4)

9.2 Show that the area of ΔPOT = $\dfrac{1}{2}a^2 \tan x$ (4)

[8]

QUESTION 10

The figure below represents a right prism with BA = BC = 5 units, $A\hat{B}C = 50°$ and $F\hat{A}C = 25°$.



10.1	Determine the area of ΔABC	(2)
10.2	Calculate the length of AC	(3)
10.3	Hence, determine the height FC of the prism.	(3)
		[8]

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $\quad r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x\ ;\ y) \rightarrow (x\cos\theta - y\sin\theta\ ;\ y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$