HILLCREST HIGH SCHOOL

MATHEMATICS

PAPER 1

TRIALS 2014

MARKS: 150

TIME: 3 Hours

This question paper consists of 7 pages and an information sheet

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions:

1. This question paper consists of 10 questions. Answer ALL the questions.
2. Show ALL calculations, diagrams, graphs etc., clearly, which you have used in determining the answers.
3. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
4. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
5. Number the answers correctly according to the numbering system used in this question paper.
6. Diagrams are NOT necessarily drawn to scale.
7. Write neatly and legibly.
8. An information sheet, with formulae, is included at the end of the question paper.

Question 1

1.1 Solve for x:

1.1.1 $x^2 = x$ (2)

1.1.2 $(x-5)(x+2) = -6$ (4)

1.1.3 $5^{x+1} + 5^x = 6$ (3)

1.1.4 $\sqrt{2x+1} - 3 = 0$ (3)

1.2 Simplify the following expression:

$$\frac{9^x . 10^{x-2}}{6^{x-1} . 15^x}$$

(5)

1.3 For which values of *x* is $\sqrt{\frac{x}{x-2}}$ real? (3)

1.4 Solve the following equation for *x* and *y*:

$y + 7 = 2x$

$x^2 - xy + 3y^2 = 15$ (5)

[25]

Question 2

The first three terms of an arithmetic sequence are:

$$2x + 1; x - 3; 3x - 4$$

2.1 Determine the value of x, and then write down the values of the first three terms. (4)

2.2 Determine the n^{th} term of the sequence. (3)

2.3. The sum of the first *n* terms of the sequence is -176. Find *n*. (4)

[11]

Question 3

3.1 Evaluate: (2)

$$\sum_{r=1}^{4} 3^{r-1}$$

3.2 Given the sequence:

1; 5; 11; 19; 29

3.2.1 Write down the next 2 terms. (2)

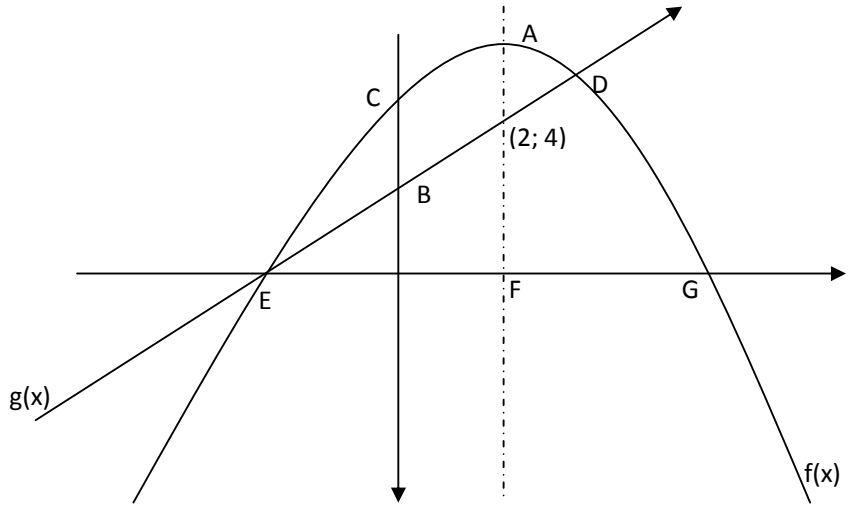
3.2.2 Find the formula for the sequence. (5)

3.3 A rubber ball is dropped vertically from a height of 15 metres and loses 20% of its previous

height at each bounce of the ball. Determine the total distance the ball travels before coming to rest. (6)

[15]

Question 4



The sketch represents the graphs of

$f(x) = -x^2 + bx + c$ and $g(x) = x + k$

The point (2; 4) lies on the axis of symmetry of f. A is the turning point of f, and B and C are respectively the points where g and f intersect the y-axis. The two graphs intersect at D and E (-2; 0)

4.1 Determine the values of *k,b* and *c*. (6)

4.2 If $f(x) = -x^2 + 4x + 12$ and $g(x) = x + 2$, determine:

4.2.1 the coordinates of A, by completing the square (5)

4.2.2 the length of BC (2)

4.2.3 the coordinates of D, the intersection of f and g (5)

[18]

Question 5

5.1 Consider the function

$$f(x) = \left(\frac{1}{3}\right)^x$$

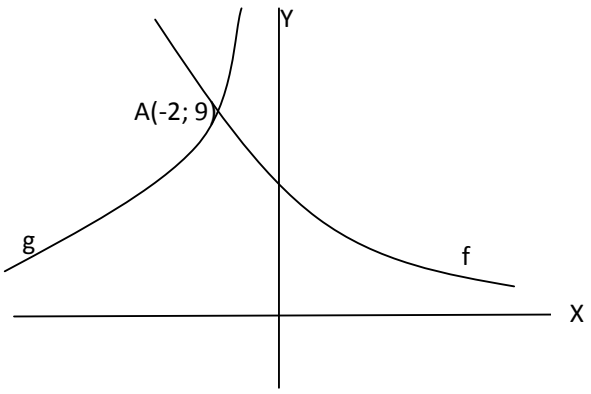
5.1.1 Is f an increasing or a decreasing function? Give a reason for your answer. (2)

5.1.2 Determine $f^{-1}(x)$ in the form y = ... (2)

5.1.3 Write down the equation of the asymptote of the graph $f(x) - 5$ (1)

5.1.4 Describe the transformation from f to g if $g(x) = log_3 x$ (2)

5.2



The figure represents the graphs of $f(x) = a^x$ and $g(x) = \frac{k}{x}, (x < 0)$

5.2.1 If point A(-2; 9) lies on both graphs, determine the values of *a* and *k.* (5)

5.2.2 If the graph h is symmetrical about the y-axis to graph g, determine the equation of h in the form *h(x)* = ... (3)

[15]

Question 6

6.1 Hillcrest High School Trust received R400 000 towards the building of a Sports Centre. The money was invested at 13,25% compounded annually. When the investment grows to one million rand, the Trust will commence the project. How many years will this take? (4)

6.2 Anne wants to buy a car for her daughter Bridget when she turns 18, by paying a fixed amount each month into a savings account, with an interest of 12% per annum compounded monthly. The daughter has just turned 8. Her mother estimates she will need R90 000 when Bridget turns 18.
Calculate how much Anne will pay each month, with the first payment being in a month's time. (5)

6.3 Rodgers Cars agrees to sell Anthony a car on a no-deposit plan provided he pays the amount owing in 10 months. The interest rate is 18% compounded monthly. Anthony works out that if he pays R6300 at the end of each month, his debt will be cleared in 10 months.

6.3.1 Calculate the cash price of the car. (4)

6.3.2 Calculate the effective interest rate that Anthony will pay. (3)

[16]

Question 7

7.1 Given $f(x) = 1 - 2x^2$, find $f'(x)$ using first principles. (5)

7.2.1 Determine $\frac{dy}{dx}$ for the following:

$$y = (x + 1)^2$$

(3)

7.2.2 Determine:

$$D_x\left[\frac{x^2-1}{2x+2}\right]$$ (4)

7.3 Given $(g)x = \frac{3x^2}{2} - 18\sqrt{x}$, calculate $g'(9)$ (5)

[17]

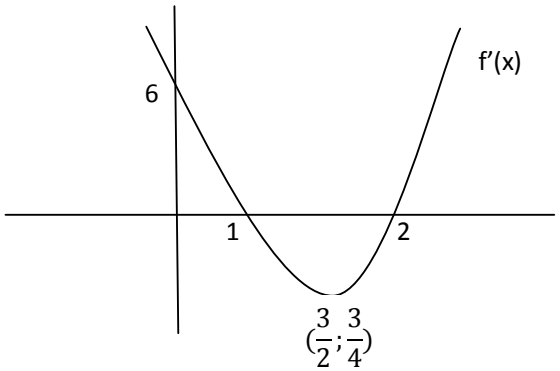
Question 8

8.1 Given $k(x) = x^3 - 3x^2 + 4$

8.1.1 Calculate the gradient of the tangent to the curve *k* at the point (3; 4). (3)

8.1.2 Find the equation of this tangent. (2)

8.2 The graph of the parabola $y = f'(x)$ is shown.



8.2.1 Write down the x-coordinate of the local minimum of y = $f(x)$ (2)

8.2.2 For which values of *x* will $f(x)$ be decreasing? (2)

8.2.3 What is the gradient of the tangent to f when *x=o*? (2)

8.2.4 At which value of *x* will there be a tangent to f parallel to the one in 8.2.3 (2)

[13]

Question 9

The height to which a plant grows during the first six months is given by the function $f(x) = 36x - 3x^2; 0 \leq x \leq 6$ where x is the age of the plant in months and $f(x)$ is the height in centimetres above the ground after x months.

9.1 What height would the plant reach after 3 months? (2)
9.2 At the end of how many months will the plant reach its maximum height? (3)
9.3 Hence, calculate the maximum height to which the plant will grow. (2)

[7]

Question 10

10.1 It is given that P(A) = 0,35; P(B) = 0,8 and P(A and B) = 0,25

10.1.1 Draw a Venn diagram to represent the above information. (4)
10.1.2 Use your Venn diagram to find the following:
a) P(A or B) (2)
b) P(A and B)' (2)

10.2 The digits 0,1,2,3,4,5 and 6 are used to make 3 digit codes.

10.2.1 How many unique codes are possible if digits can be repeated? (1)
10.2.2 How many unique digit codes are possible if the digits cannot be repeated? (1)
10.2.3 In the case where digits can be repeated how many codes are numbers that are greater than 300 and exactly divisible by 5? (3)

[13]

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 \quad 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x\,;\,y) \to (x\cos\theta - y\sin\theta\,;\, y\cos\theta + x\sin\theta)$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$