

Basic Education

KwaZulu-Natal Department of Basic Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

PREPARATORY EXAMINATION

SEPTEMBER 2015

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MARKS: 150

TIME: 3 hours

**This question paper consists of 12 pages,
1 information sheet and an answer book of 19 pages.**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.

QUESTION ONE

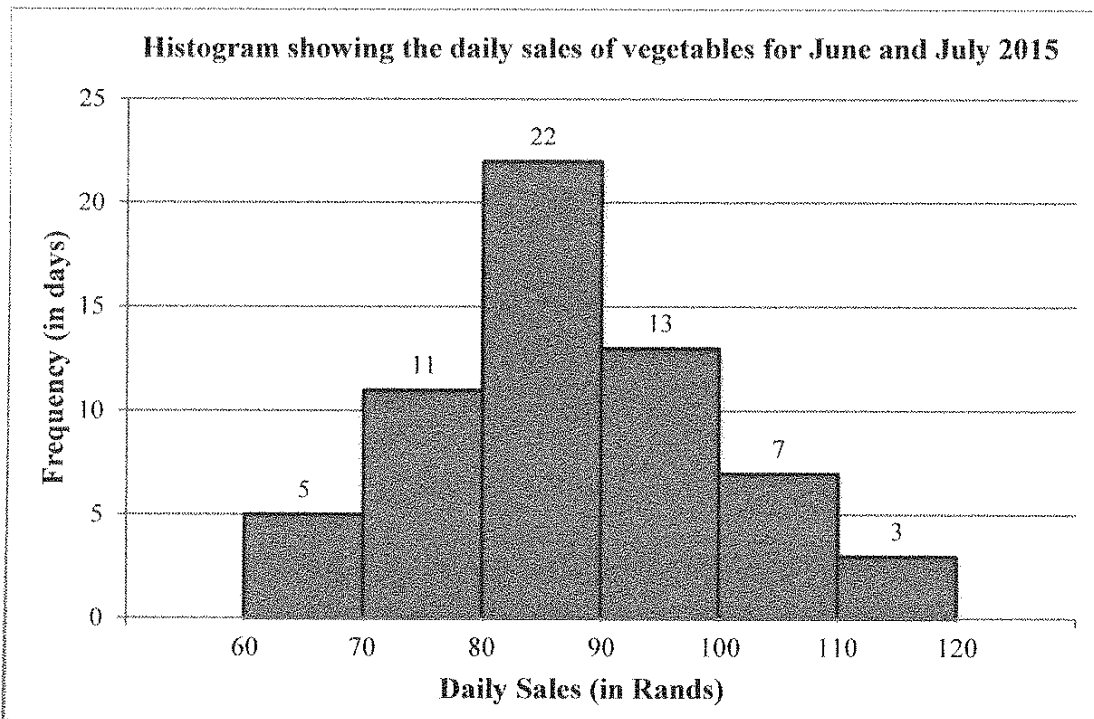
Ten athletes took part in a javelin throwing competition. Their height, in *cm*, and their arm span, in *cm*, is shown in the table below.

Athlete	1	2	3	4	5	6	7	8	9	10
Height (in <i>cm</i>)	156	173	181	174	167	170	169	174	177	168
Arm span (in <i>cm</i>)	164	181	193	178	172	178	165	183	190	173

- 1.1 Represent the height and arm span for each athlete on the scatter plot provided in the answer book. (3)
- 1.2 Determine the equation of the least squares regression line. (4)
- 1.3 Use the equation in 1.2 to estimate the arm span of an athlete whose height is 176 *cm*. (2)
- 1.4 The correlation coefficient for this set of data is 0,89. Comment on the strength of the relationship between height and arm span. (1)
- [10]**

QUESTION TWO

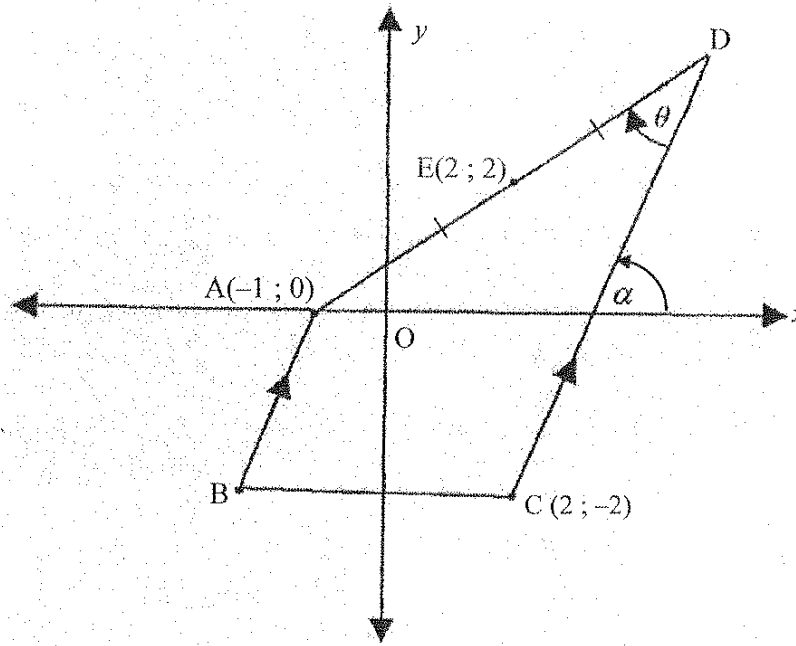
Mrs. Panyane, a street vendor, has kept record of the daily sales of vegetables for the months of June and July 2015. The information is represented in the histogram below.



- 2.1 Use the information provided in the histogram to complete the frequency and cumulative frequency columns in the table provided in the answer book. (3)
 - 2.2 Use the grid provided in the answer book to draw an ogive (cumulative frequency graph) for the daily sales over this period. (4)
 - 2.3 Use your cumulative frequency graph to estimate the median value for the daily sales. (2)
 - 2.4 Estimate the interval of the upper 25% of the daily sales. (1)
- [10]**

QUESTION THREE

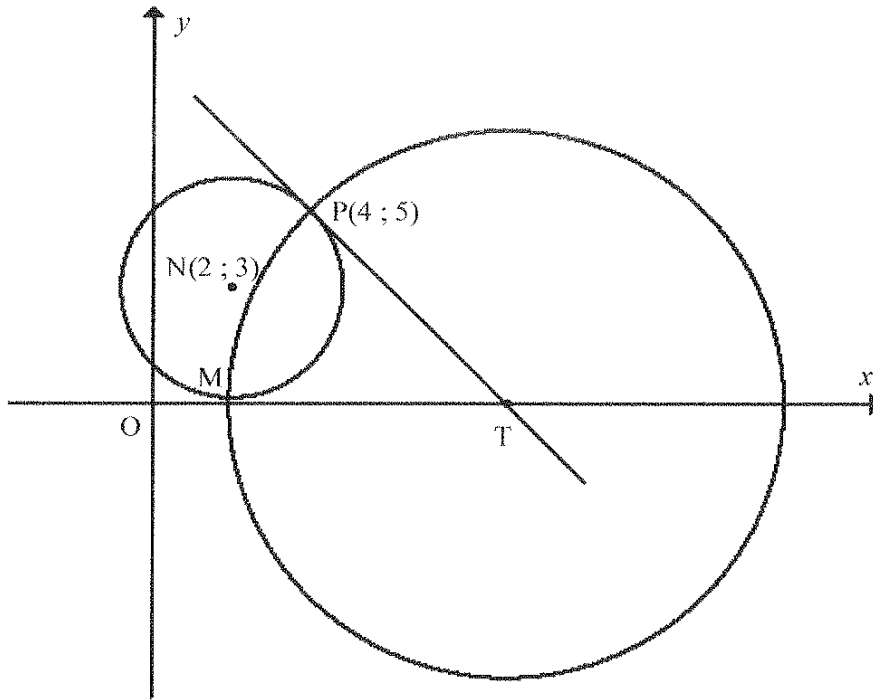
In the diagram below, $A(-1; 0)$, B , $C(2; -2)$ and D are the vertices of a trapezium having $AB \parallel DC$. The length of DC is three times the length of AB (i.e. $DC = 3AB$). $\hat{ADC} = \theta$. $E(2; 2)$ is the midpoint of AD . The angle of inclination of DC is α .



- 3.1 Determine the coordinates of D . (2)
- 3.2 Calculate the size of α , correct to ONE decimal place. (3)
- 3.3 Determine the equation of AB in the form $y = mx + c$. (3)
- 3.4 Calculate the size of θ , correct to ONE decimal place. (3)
- 3.5 Calculate the coordinates of B . (5)
- [16]**

QUESTION FOUR

In the figure below, T is a point on the x-axis. A circle having T as its centre intersects a circle having N(2 ; 3) as its centre at P(4 ; 5) and M. TP is a tangent to the circle centre N at P.



- 4.1 Determine the equation of circle centre N in the form: $(x - a)^2 + (y - b)^2 = r^2$. (4)
 - 4.2 Calculate the coordinates of T, the x-intercept of PT. (6)
 - 4.3 Calculate the length of PT. Leave your answer in surd form. (2)
 - 4.4 Calculate the area of circle centre T. Give your answer rounded off to the nearest integer. (2)
 - 4.5 Calculate the size of \hat{NTP} , correct to ONE decimal place. (2)
 - 4.6 Prove that MNPT is a kite. (3)
 - 4.7 Calculate the size of \hat{MNP} , correct to ONE decimal place. (4)
- [23]**

QUESTION FIVE

5.1 Simplify to a single trigonometric ratio of A:

$$\frac{\tan(180^\circ + A) \cdot \cos(180^\circ - A) \cdot \sin(360^\circ - A)}{\cos(90^\circ - A)} \quad (6)$$

5.2 If $\cos 26 = r$, determine the following in terms of r , in its simplest form:

5.2.1 $\cos 52^\circ$ (3)

5.2.2 $\tan 71^\circ$ (6)

5.3 Prove the identity: $\frac{\sin 2x}{\cos 2x + \sin^2 x} = 2 \tan x$ (4)

[19]

QUESTION SIX

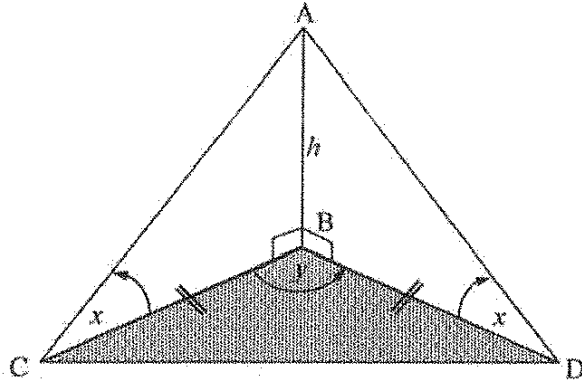
6.1 Determine the general solution of: $\cos 2x = \sin(x - 30^\circ)$. (7)

6.2 On the set of axes provided in the answer book, draw the sketch graphs of $f(x) = \cos 2x$ and $g(x) = \sin(x - 30^\circ)$ for $x \in [-180^\circ; 90^\circ]$. Clearly indicate the coordinates of the turning points and the intercepts with the axes. (6)

6.3 Write down the values of x for which $g(x) > f(x)$ in the given interval. (3)
[16]

QUESTION SEVEN

In the diagram, C and D represent two ships in the same horizontal plane as B, the base of a lighthouse AB which is h metres high. Each ship is the same distance away from the base of the lighthouse. The angles of elevation from C and D to A are both x . $\hat{C}BD = y$.



7.1 Write BD in terms of h and a trigonometric ratio of x . (2)

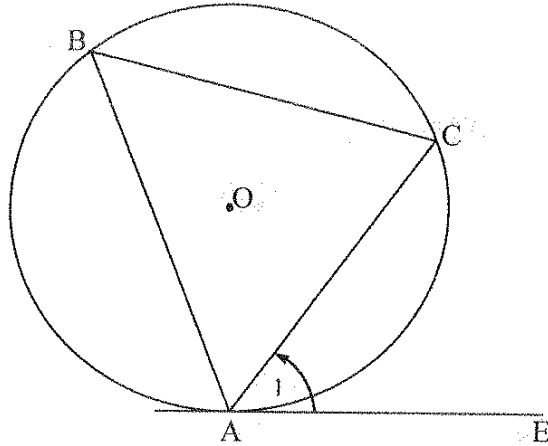
7.2 Hence or otherwise prove $CD^2 = \frac{2h^2(1 - \cos y)}{\tan^2 x}$ (4)

[6]

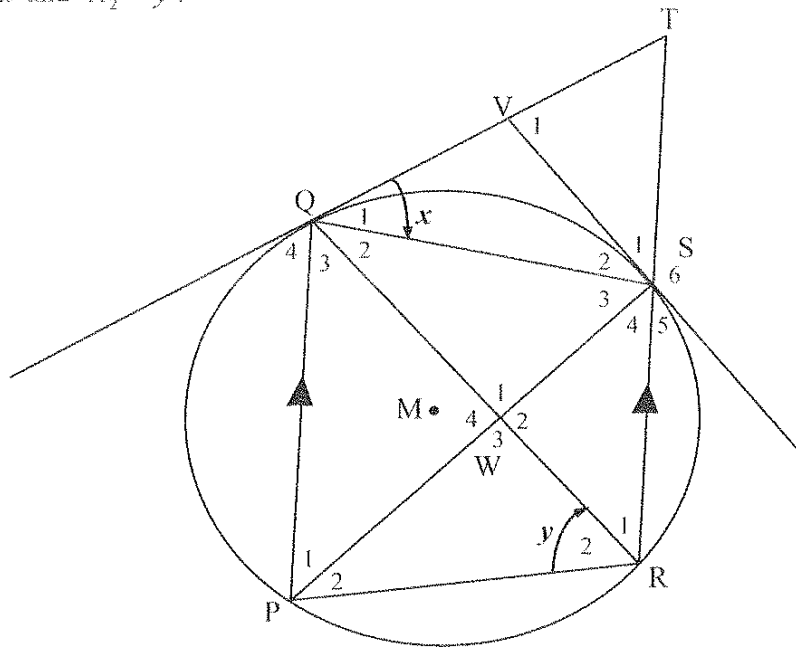
NOTE: Give reasons for your statements in these questions 8, 9 and 10.

QUESTION EIGHT

- 8.1 In the diagram below, O is the centre of the circle passing through A , B and C . EA is a tangent to the circle at A . Use this diagram to prove the theorem which states that $\hat{EAC} = \hat{ABC}$. (6)



8.2 In the diagram below, PQ and RS are chords of the circle such that $PQ \parallel RS$. The tangent to the circle at Q meets RS produced at T and the tangent at S meets QT at V. PS and QR intersect at W. QS and PR are drawn. Let $\hat{Q}_1 = x$ and $\hat{R}_2 = y$.



8.2.1 Write down a reason why $QV = VS$. (1)

8.2.2 Write down the following angles in terms of x :

(a) \hat{S}_2 (2)

(b) \hat{R}_1 (2)

(c) \hat{V}_1 (3)

8.2.3 Show that $\hat{R}_1 = \hat{S}_4$ (4)

8.2.4 Prove that QVSW is a cyclic quadrilateral. (4)

8.2.5 Write down the following angles in terms of y :

(a) \hat{Q}_4 (2)

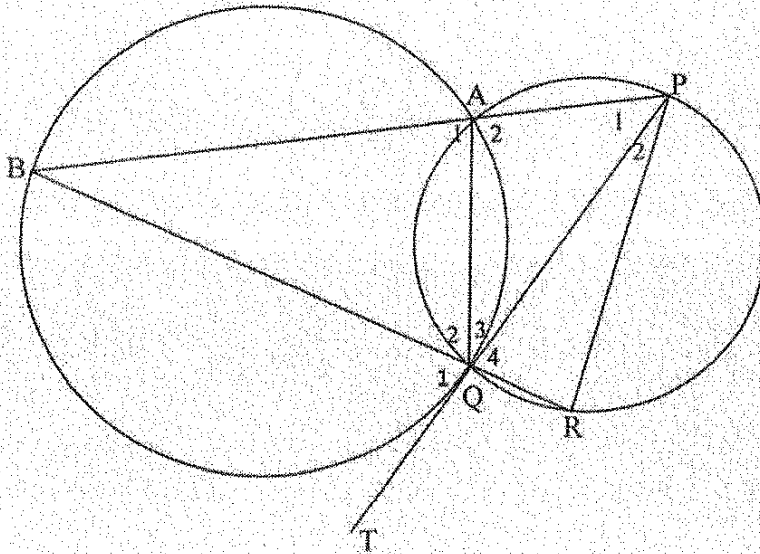
(b) \hat{T} (2)

8.2.6 If M is the centre of the circle, hence prove that PMWR is a cyclic quadrilateral. (2)

[28]

QUESTION NINE

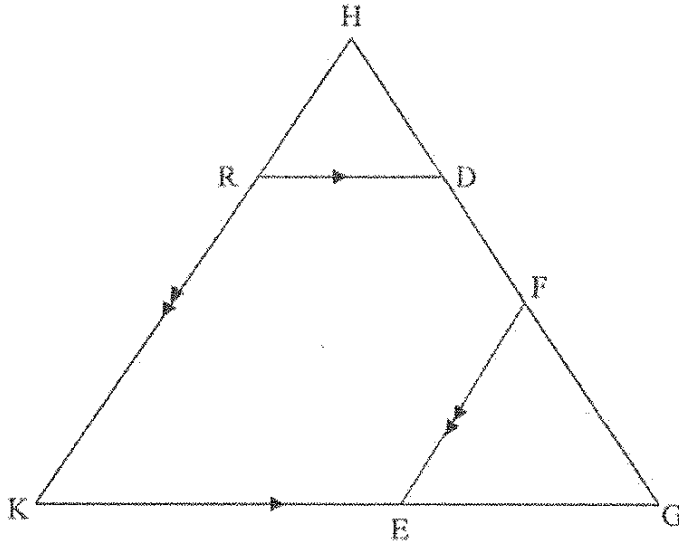
In the diagram below, PQT is a tangent to the larger circle ABQ at Q . A smaller circle intersects the larger circle at A and Q . BAP and BQR are straight lines with P and R on the smaller circle. AQ and PR are drawn.



- 9.1 Prove that $PQ = PR$ (7)
 - 9.2 Prove that $\triangle PBQ \sim \triangle PQA$. (4)
 - 9.3 Prove that the lengths of PA , PR and PB (in this order) form a geometric sequence. (3)
- [14]**

QUESTION TEN

In the diagram below, $\triangle GHK$ is drawn having the point R on KH and the points D and F on GH such that $RD \parallel KG$ and $EF \parallel KH$. It is also given that $RH = 3$ units, $RK = 9$ units, $HD = 2$ units and $GE : EK = 1 : 2$.



Calculate the lengths of:

10.1 DG (3)

10.2 FD. (5)
[8]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS
INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$