# Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

**MATHEMATICS P2**

**PREPARATORY EXAMINATION**

**SEPTEMBER 2017**

**NATIONAL SENIOR CERTIFICATE**

**GRADE 12**

**MARKS: 150**

**TIME: 3 hours**

**This question paper consists of 12 pages, including information sheet.**

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## QUESTION 1

The table below shows the heights of palm trees in a park.

| HEIGHT IN CM | NUMBER OF PALM TREES |
|---|---|
| $120 < x \leq 135$ | 1 |
| $135 < x \leq 150$ | 15 |
| $150 < x \leq 165$ | 45 |
| $165 < x \leq 180$ | 28 |
| $180 < x \leq 195$ | 1 |

1.1 Determine the estimated mean height of the palm trees in the park. (2)

1.2 Draw an ogive to represent this data. (4)

1.3 Use your ogive curve to estimate the:

1.3.1 median height of the palm tree. (2)

1.3.2 interquartile range (IQR). (3)

**[11]**

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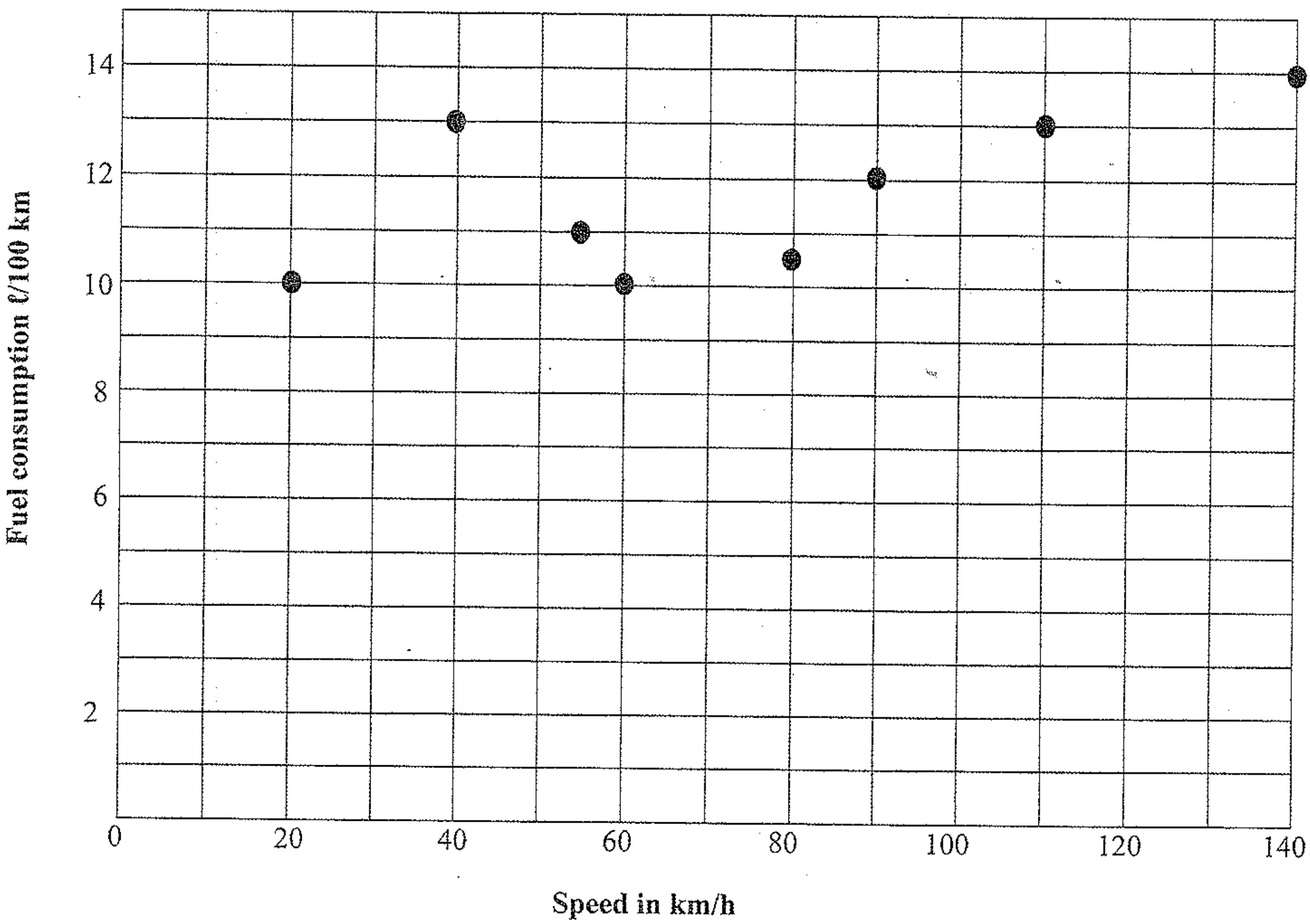
## QUESTION 2

The scatter plot below shows the fuel consumption versus the speed of a motor car.




2.1 Identify an outlier. Write down its co-ordinates. (1)

2.2 Determine:

2.2.1 the equation of the regression line excluding the outlier. (3)

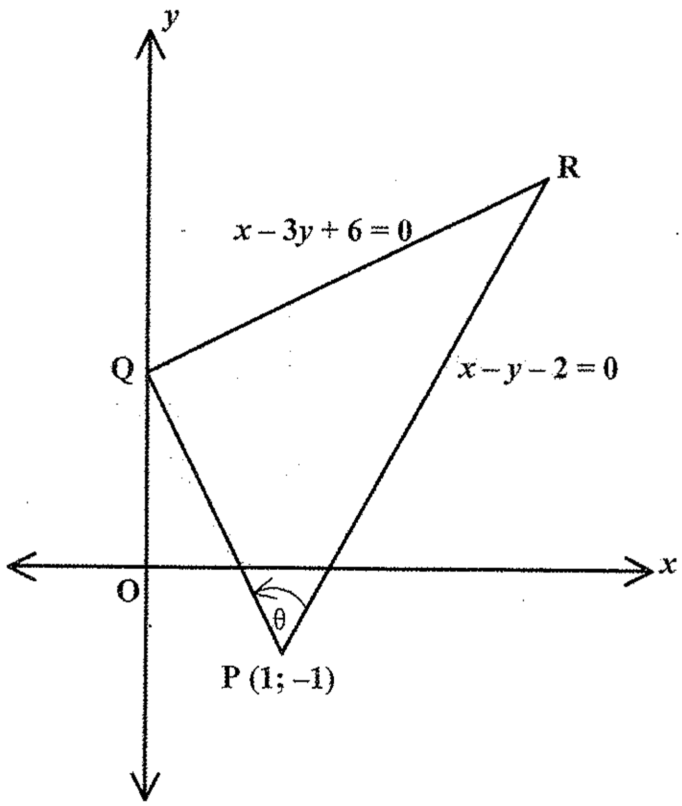
2.2.2 the correlation coefficient excluding the outlier and explain the type of correlation. (2)

2.2.3 the average fuel consumption of the motor car. (2)

**[8]**

## QUESTION 3

In the figure below, PQR is a triangle with P(1 ; −1) . Q is a point on the $y$-axis. The equations of QR and PR are $x-3y+6=0$ and $x-y-2=0$ respectively. Given $Q\hat{P}R=\theta$.




3.1 Show that the co-ordinates of Q are (0 ; 2). (2)

3.2 Write down the gradient of QR. (2)

3.3 Prove that $P\hat{Q}R=90^\circ$. (2)

3.4 Calculate the co-ordinates of R. (3)

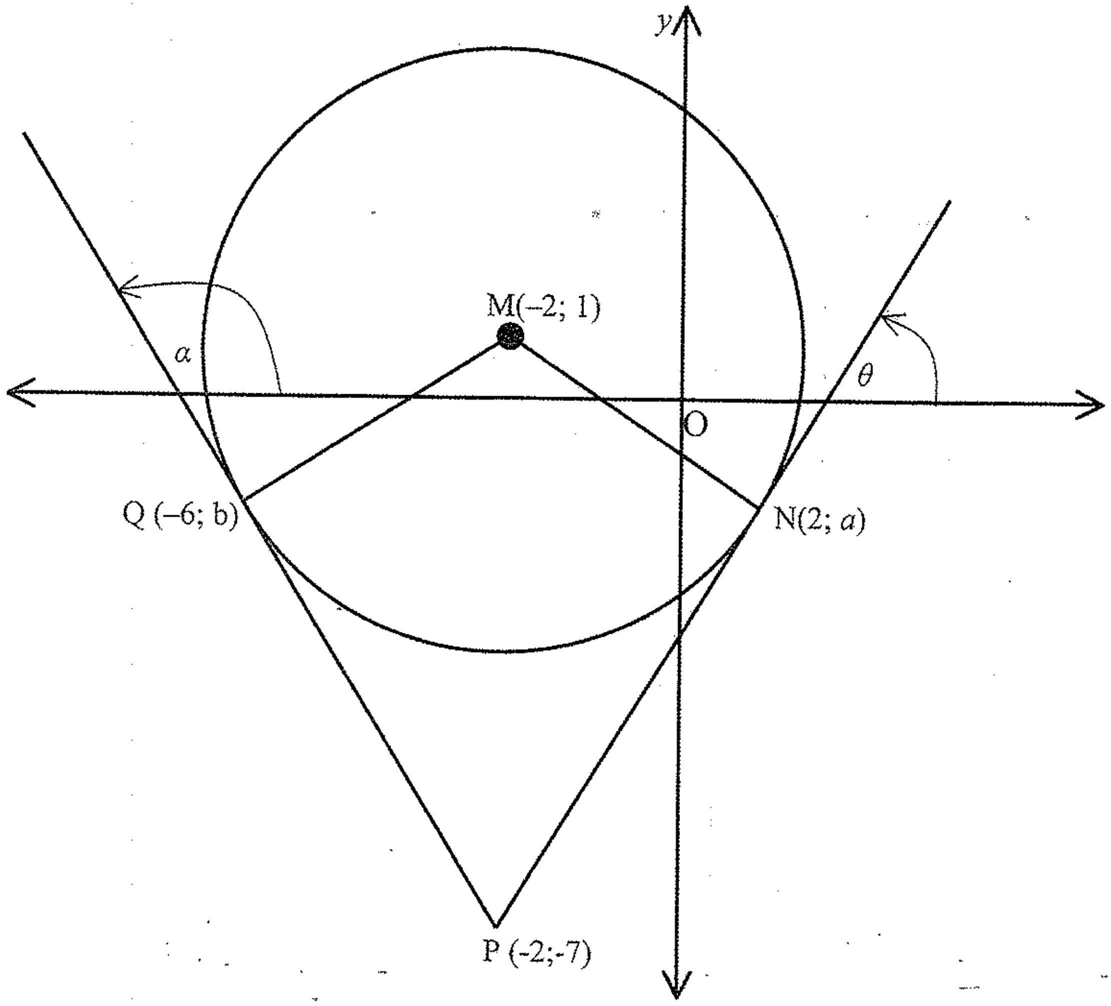
3.5 Calculate the area of Δ PQR. (4)

3.6 Calculate the length of PR. (leave your answer in the simplest surd form). (2)

**[15]**

## QUESTION 4

4.1 In the diagram below, MN is a radius of a circle with centre M(–2 ; 1). The co-ordinates of N are (2 ; $a$) and $a < 0$. The co-ordinates of P are (-2'-7). PQ and PN are tangents to the circle at Q and N respectively.
The coordinates of Q is (–6 ; $b$). PM is parallel to the $y$ – axis.




4.1.1 Deduce that $a = -3$. Show all your workings. (5)

4.1.2 Determine the equation of the circle. (2)

4.1.3 Calculate the gradient of the tangents at Q and N. (4)

4.1.4 If the angle of inclination of lines PN and PQ are θ and α respectively, without using a calculator, show that $\tan^2 \alpha + \tan^2 \theta = 2$. (4)

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4.2 The circle defined by $(x+1)^2 + (y-1)^2 = 16$ has centre C and circle defined by $x^2 + y^2 - 2y = 8$ has centre D.

4.2.1 Show that the two circles touch each other internally. (5)

4.2.2 Determine the equation of the common tangent to the circles. (2)
**[22]**

## QUESTION 5

5.1 Show, without using a calculator, that

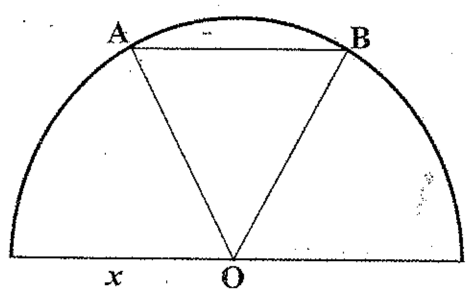
$$\sqrt{2}\cos(-45^\circ) + \cos 210^\circ - \tan 840^\circ = \frac{2+\sqrt{3}}{2}.$$ (5)

5.2 If $\sin\theta = \dfrac{2n}{n^2+1}$, $n > 1$ and $0^\circ < \theta < 90^\circ$, prove that $\dfrac{1+\sin\theta}{\cos\theta} = \dfrac{n+1}{n-1}$. (7)

5.3 Prove the identity:

$$\frac{\sin 2x}{\cos x\,(1-\cos 2x)\left(1+\dfrac{1}{\tan^2 x}\right)} = \sin x$$ (5)

5.4 In the figure below, semi-circle with centre O has radius $x$. Points A and B are on the circumference of circle. Calculate in terms of $x$ the maximum area of $\Delta$AOB.




(3)

**[20]**

## QUESTION 6

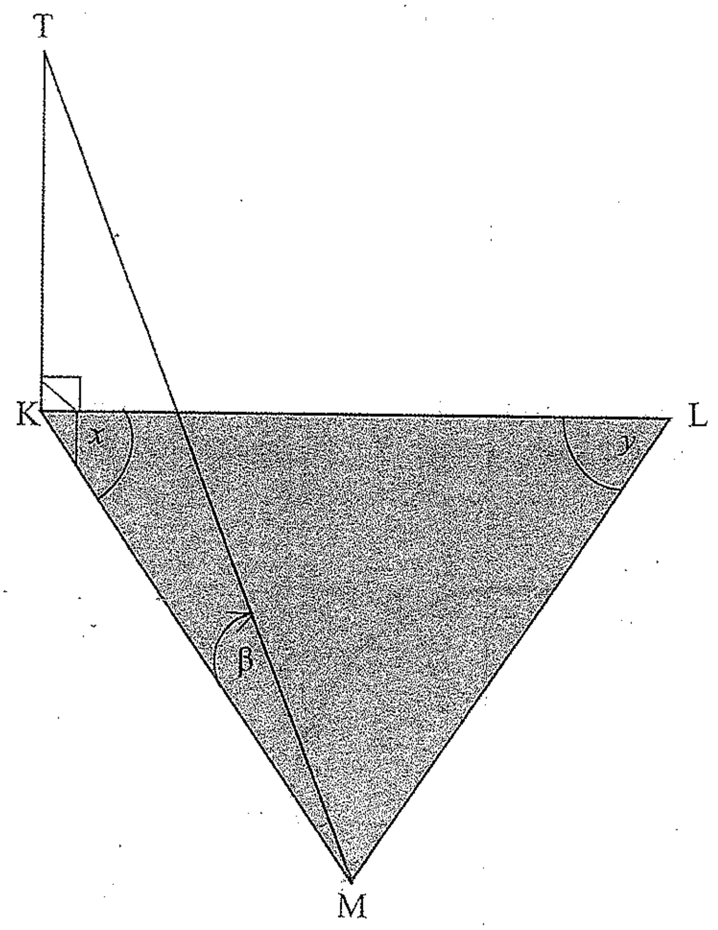
6.1 6.1.1 Write down an expansion for $\sin(x + 30^\circ)$. Leave your answer in surd form. (3)

6.1.2 Hence, solve the equation:

$$2\cos x = \sin(x + 30^\circ) \text{ for } x \in [-180^\circ; 270^\circ]$$ (7)

6.2 On the axes provided, in your answer book sketch the graphs of $f(x) = 2\cos x$ and $g(x) = \sin(x + 30^\circ)$ for the interval $x \in [-180^\circ ; 270^\circ]$. (6)

6.3 TK is a pole with K in the same horizontal plane as L and M. The angle of elevation of T from M is $\beta$. $L\hat{K}M = x$ and $K\hat{L}M = y$.

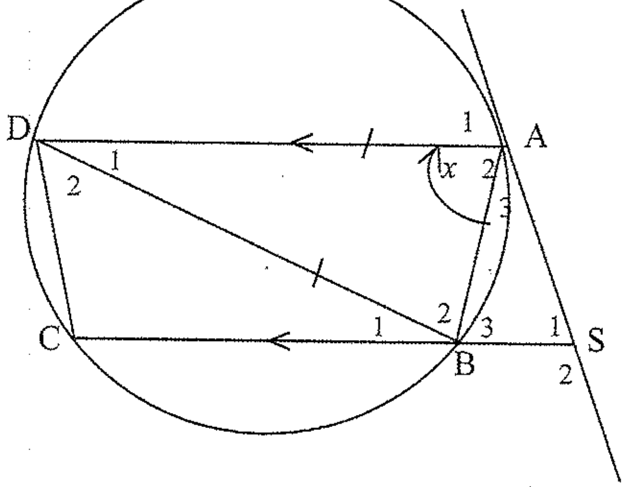



Show that $KT = \dfrac{KL \sin y \,.\, \tan\beta}{\sin(x + y)}$ (5)

**[21]**

## QUESTION 7

7. Refer to the figure below. ABCD is a cyclic quadrilateral. AS is a tangent to the circle at A. CB is produced to S. AD || SBC; AD = BD; $\hat{A}_2 = x$.

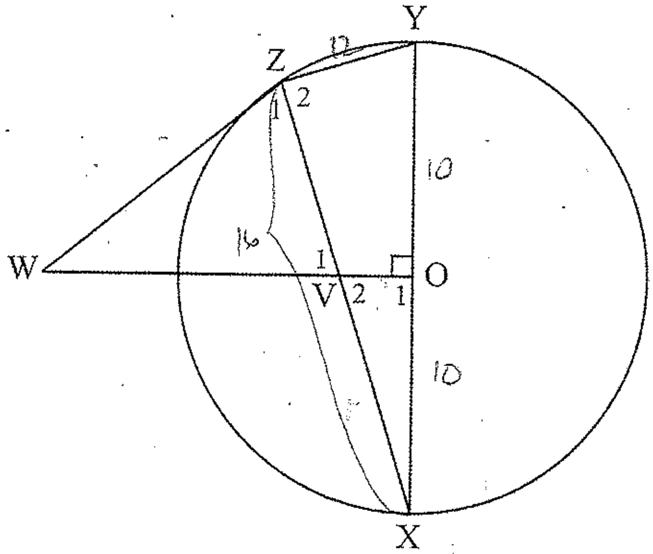



Write down, with reasons, FIVE other angles each equal to $x$. (9)

**[9]**

## QUESTION 8

In the figure below, O is the centre of the circle ZYX. WO intersects XZ at V and WZ is a tangent to the circle at Z. WO ⊥ XY.




8.1 Prove that VOYZ is a cyclic quadrilateral. (3)

8.2 Prove that ΔWVZ is isosceles. (3)

8.3 Prove that ΔXOV ||| ΔXZY. (4)

8.4 Calculate VO, if XZ = 16 units, ZY = 12 units and the radius of the circle is 10 units. (3)

**[13]**

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## QUESTION 9

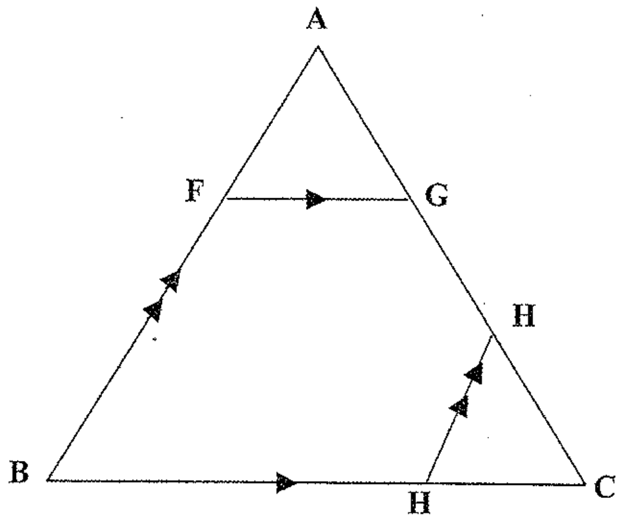
In the diagram below FG || BC, HJ || AB.
FA = 3 units
FB = 9 units
AG = 2 units

CJ: JB = 1:3

Calculate (stating reasons) the lengths of:

9.1 GC (4)

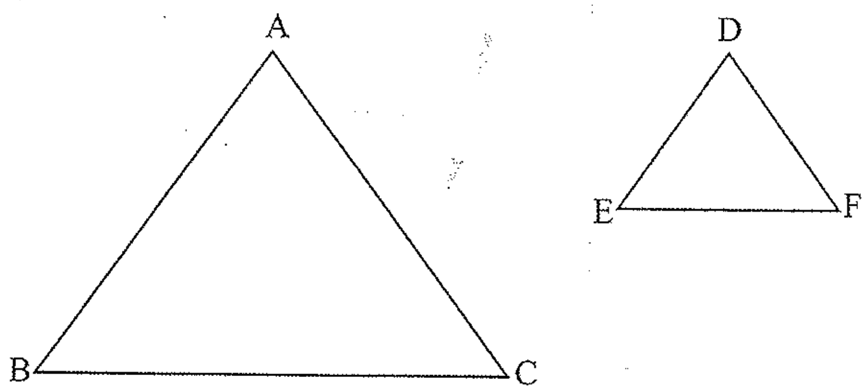
9.2 GH (5)




[9]

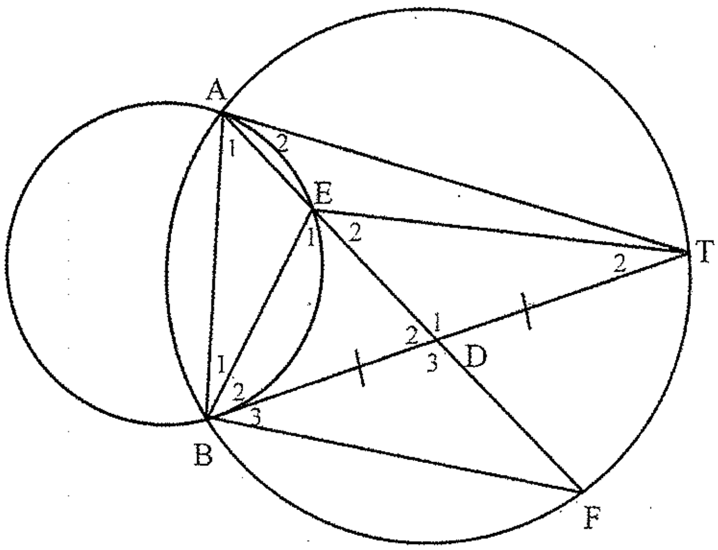
## QUESTION 10

10.1 ΔABC and ΔDEF with $\hat{A} = \hat{D}$; $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$ are shown below.




Prove that: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$ (7)

10.2 In the figure below, two circles intersect at A and B. TB is a tangent to the smaller circle at B. The line through D and A cuts the circles at E and F such that BD = DT. AB, BE and EA are joined.




10.2.1 Prove that $\Delta TDA \parallel\!| \Delta FDB$. (4)

10.2.2 Prove that $TB^2 = 4FD \cdot AD$. (2)

10.2.3 Prove that $BD^2 = DE.AD$. (4)

10.2.4 Deduce that ET = BF. (5)

**[22]**

**TOTAL MARKS: 150**