HILLCREST HIGH SCHOOL

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INTERNAL EXAMINATIONS

GRADE 12

MATHEMATICS

PAPER 1

TRIALS EXAMINATION

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of **12** questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Write neatly and legibly.

QUESTION 1

1.1 Solve for x :

1.1.1 $5^{x+1}(x+5)=0$ (1)

1.1.2 $4x+5=2x^2$, correct to TWO decimal places. (4)

1.1.3 $-x^2+16\geq 0$ (3)

1.1.4 $4\sqrt{x}-4x+3=0$ (5)

1.2 Solve simultaneously for x and y :

$4^{x+y}=2^{y+4}$ and $2x^2-3xy=-4$ (6)

1.3 Solve for x if $(k+2)$ is a factor of $k^{50}-x^{25}$. (3)

[22]

QUESTION 2

The first three terms of an arithmetic sequence are -6 ; -11 and -16.

2.1 Write down the next TWO terms of this sequence. (2)

2.2 Determine the general term (T_n) of the sequence. (2)

2.3 Consider the terms in the sequence which are divisible by 3.
Calculate the value of the sum of the first 16 terms which are divisible by 3. (4)

2.4 The sequence -6 ; -11 and -16 forms the first differences of a new sequence with a second term of 0. Determine a formula for the n^{th} term of this new sequence. (4)

[12]

QUESTION 3

3.1 Write down a formula for the sum of the first n terms of a geometric series. (1)

3.2 Given that $(k+1)$; $(k-1)$ and $(2k-5)$ are the first three terms of a converging geometric series. Calculate:

3.2.1 the value(s) of k. (6)

3.2.2 the sum of the infinite series, given that $k=3$. (3)

3.3 Determine the value of :

3.3.1 $\sum_{k=1}^{12} \frac{1}{2}(7-k)$ (2)

3.3.2 $\sum_{p=1}^{\infty} \frac{5}{8}.\left(\frac{1}{3}\right)^{p-1}$ (3)

3.3.3 Hence, or otherwise, determine the value of *n* if it is given that:

$$\sum_{r=1}^{n} 4\left(\frac{1}{2}\right)^{r} = \sum_{p=1}^{\infty} \frac{5}{8}.\left(\frac{1}{3}\right)^{p-1} + \sum_{k=1}^{12} \frac{1}{2}(7-k)$$ (3)

[18]

QUESTION 4

The curve of the following function passes through the point $(4;-2)$: $f(x) = \log_a x$; $a > 0$

4.1 Prove that $a = \frac{1}{2}$. (2)

4.2 Determine the equation of g if the curve of g is a reflection of the curve of f in the y–axis. (2)

4.3 Determine the equation of f^{-1} , the inverse of f , in the form $f^{-1}(x) = ...$ (2)

4.4 If the graph of f^{-1} is shifted 2 units to the right and 1 unit down to give the function h , write down the equation of h . (2)

[8]

QUESTION 5

The graph of an increasing exponential function with equation $h(x) = a.b^x + q$ has the following properties:

- Range: $y > -3$
- The points $(0; -2)$ and $(1; -1)$ lie on the graph of h .

5.1 Determine the equation that defines *h.* (4)

5.2 Draw a neat sketch graph of h , indicating the asymptote and *y*–intercept with the axes. (3)

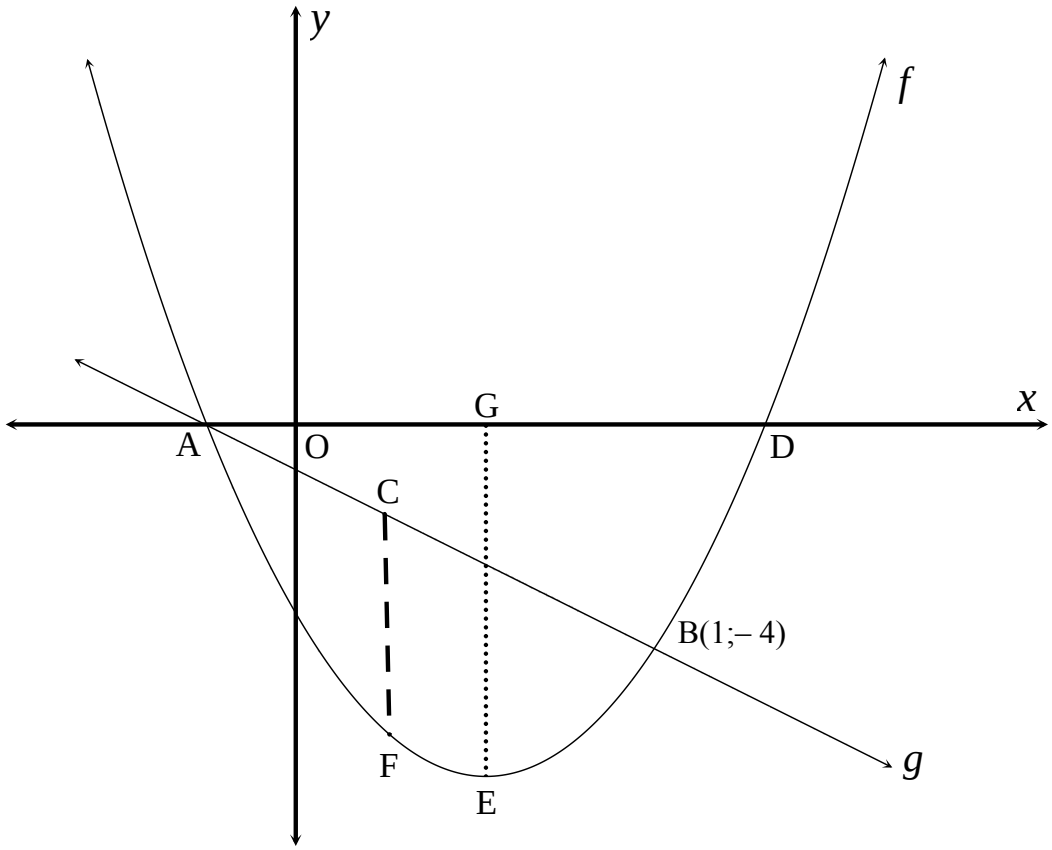
[7]

QUESTION 6

The sketch shows the graph of a parabola defined by $f(x) = 2x^2 - 2x - 4$ with turning point E.

The straight line $g(x) = mx + k$ cuts the parabola at A and B $(1; -4)$.

GE is parallel to the *y*–axis.



6.1 Determine the length of AD and GE. (3)

6.2 Show that the equation of the straight line represented by $g(x)$ is given by $g(x) = -2x - 2$. (2)

6.3 Determine the value of m such that the line $y = 3x + m$ is a tangent to the graph of $f(x)$. (5)

6.4 Calculate the maximum length of CF where CF is a line segment parallel to the *y*–axis with C on the graph of $g(x)$ and F on the graph of $f(x)$. (3)

[13]

QUESTION 7

Consider the functions f and g where: $f(x)=3x-1$ and $g(x)=x-1$.

It is also given that: $h(x)=\dfrac{f(x)}{g(x)}$; $x\neq 1$

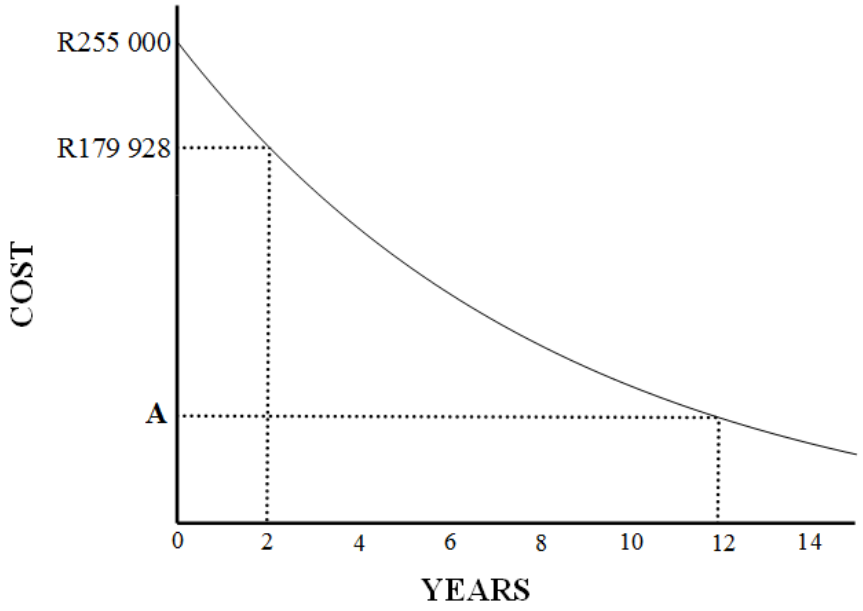
7.1 Show that $h(x)$ can be written in the form $y=\dfrac{a}{x-p}+q$, where $a=2;\ p=1$ and $q=3$. (3)

7.2 Draw a neat sketch of $h(x)$ showing all intercepts with the axes as well as the any asymptotes. (3)

[6]

QUESTION 8

8.1 The graph below shows the depreciating value of a car over a period of time (in years):



8.1.1 What is the cost of the new car? (1)

8.1.2 What type of depreciation is illustrated? (1)

8.1.3 Use the information on the graph to find the rate of depreciation. (3)

8.1.4 Calculate **A** , the value of the car after 12 years. (2)

8.2 A student decides to save R300 at the end of every month for the next three years, and invests the money in a savings account with a 11,8% per annum interest rate compounded monthly.

Determine how much money she will have at the end of the third year. (3)

8.3 Mr Jones takes out a bank loan of R 260 000 to be paid back over 20 years at an interest rate of 9,2% p.a. compounded monthly.

8.3.1 Calculate the value of his monthly payments. (3)

8.3.2 How much money does he still owe the bank after 12 years? (3)

[16]

QUESTION 9

9.1 If $f(x) = 2x - x^2$, determine $f'(x)$ from FIRST PRINCIPLES. (5)

9.2 Determine:

9.2.1 $\frac{dy}{dx}$ if $xy = 3$ (2)

9.2.2 $D_x\left[\frac{\sqrt[3]{x^2} - 2x + \pi}{x^2}\right]$ (4)

9.3 Draw a rough sketch of $g(x) = ax^3 + bx^2 + cx + d$ where :

- $a > 0$
- $g'(x) > 0$ when $x < -1$ and $x > \frac{5}{3}$
- $g'(-1) = 0$
- one of the $x-$intercepts is -1
- $y-\text{intercept} = -3$

(4)

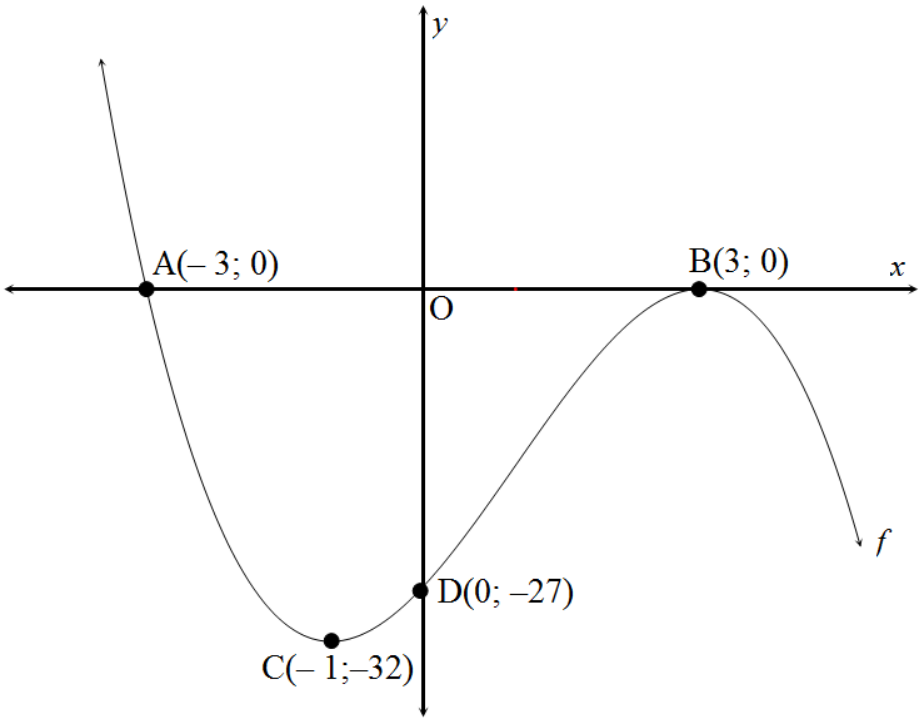
[15]

QUESTION 10

The graph of $f(x) = -x^3 + 3x^2 + 9x - 27$ is sketched below.

The graph of f intersects the x–axis at A and B, and the y–axis at D.

B and C are turning points of f.



10.1 For what value(s) of x is f strictly increasing. (1)

10.2 Calculate the average gradient of f between A and C. (2)

10.3 Show that f has a point of inflection when $x = 1$. (2)

10.4 Using the graph, state the solutions to $f'(x) = 0$. (2)

10.5 Write down the equation of g(x), the reflection of f over the x–axis. (1)
[8]

QUESTION 11

A bucket has two pipes entering it. One is **filling** the tank at a variable rate while the other is **draining** it at a variable rate.

The volume of water (in litres) in the tank at time t (in hours) with $t \in [0;3]$ is given by:
$V(t) = -t^3 - 2t^2 + 15t$.

11.1 What is the average rate of flow in litres/hour in the first hour? (2)

11.2 Did the volume of water increase or decrease over that time? (1)

11.3 What is the instantaneous rate of flow at 2 hours? (2)

11.4 At what point in the three hour time interval was the bucket fullest? Give your answer to the nearest minute. (2)

11.5 What is the maximum volume the bucket contained? Give your answer to the nearest litre. (1)

[8]

QUESTION 12

12.1 A survey was conducted among 100 boys and 60 girls to determine how many of them watched TV in the period during which examinations were written.

Their responses are shown in the partially completed table below.

	Watched TV during examinations	Did not watch TV during examinations	Totals
Male	80	**a**	
Female	48	12	
Totals	**b**	32	160

12.1.1 Calculate the values of **a** and **b**. (2)

12.1.2 Are the events 'being a male' and 'did not watch TV during examinations' mutually exclusive? Give a reason for your answer (2)

12.1.3 If a learner who participated in this survey is chosen at random, what is the probability that the learner:

(a) Watched TV in the period during which the examinations were written? (1)

(b) Is not a male and did not watch TV in the period during which examinations were written? (2)

12.2 Consider the following single digits:

1	**2**	**3**	**5**	**6**

12.2.1 How many three-digit codes can be formed with the digits above if you are allowed to use each digit more than once? (1)

12.2.2 How many four-digit codes greater than 2 200 can be created with the digits above if you may use each digit only once? (4)

12.3 The letters of the word 'SYLLABUS' are used to form different eight letter codes.

Determine the probability that the code formed will start and end with the same letters. Leave your answer as a simplified common fraction. (5)

[17]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \; ; \; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right) \qquad m = \tan\theta$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\qquad a^2 = b^2 + c^2 - 2bc.\cos A$

$$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha+\beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha-\beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha-\beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum (x-\bar{x})(y-\bar{y})}{\sum (x - \bar{x})^2}$$