HILLCREST HIGH SCHOOL

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INTERNAL ASSESSMENT

GRADE 12

MATHEMATICS
Paper 1
SEPTEMBER 2020

MARKS: **150**

TIME: **3 hours**

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round answers off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet, with formulae, is included at the end of the question paper.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write legibly and present your work neatly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(x-3)(2x+1)=0$ (2)

1.1.2 $5x(x+1)+1=0$ (correct to TWO decimal places) (3)

1.1.3 $x^2-x\geq 6$ (3)

1.1.4 $x-3\sqrt{x-6}=4$ (4)

1.2 Solve simultaneously for x and y: $x^2+y^2-25=0$ and $x-2y=5$ (5)

1.3 If $3^{x+2}=6^x$, prove that $x=\log_2 9$. (3)

[20]

QUESTION 2

2.1 Given the sequence: $m-3;\ -3;\ m+1;\ m;\ ...$ with a constant second difference.

2.1.1 Determine the value of m. (3)

2.1.2 If $m=-3$, determine the n^{th} term of this sequence. (4)

2.2 The sum of the first n terms of an arithmetic sequence is given by: $S_n=\frac{5}{2}n^2+\frac{7}{2}n$

Determine:

2.2.1 The first term. (2)

2.2.2 The common difference. (3)

2.2.3 T_{10}. (2)

2.3 Find the value of n if: $\sum_{k=1}^{n}3^k=1092$ (4)

[18]

QUESTION 3

Andrew saved R550 during the first month of his working life. In each subsequent month, he saved 8,5% more than what he had saved in the previous month.

3.1 Calculate how much Andrew saved in the 8^{th} working month. (3)

3.2 How much did he save altogether in his first 8 working months? (3)

3.3 In which month of his working life did he save more than R2 000 for the first time? (4)
[10]

QUESTION 4

Given the function defined by: $f(x) = -2(x-3)^2 + 8$

4.1 Write down the coordinates of the turning point of *f*. (2)

4.2 Determine the coordinates of the x and y-intercepts of *f*. (3)

4.3 Draw sketch graph of *f*, indicating all intercepts with the axes and the turning point. (3)

4.4 Use your drawn sketch graph in QUESTION 4.3 to write down the values of x for which $f(x) < 0$. (2)

4.5 Show that there is no real number x that satisfies the equation $k(x) = 0$ if $k(x)$ is $f(x)$ shifted 9 units down. (3)
[13]

QUESTION 5

Given the function defined by: $p(x) = \left(\frac{1}{2}\right)^x - 4$

5.1 Determine the x and y-intercepts of *p*. (3)

5.2 Write down the equation of the asymptote of *p*. (1)

5.3 Draw sketch graph of *p*. Clearly indicate all intercepts with the axes and the asymptote. (3)

5.4 Write down the range of *p*. (1)

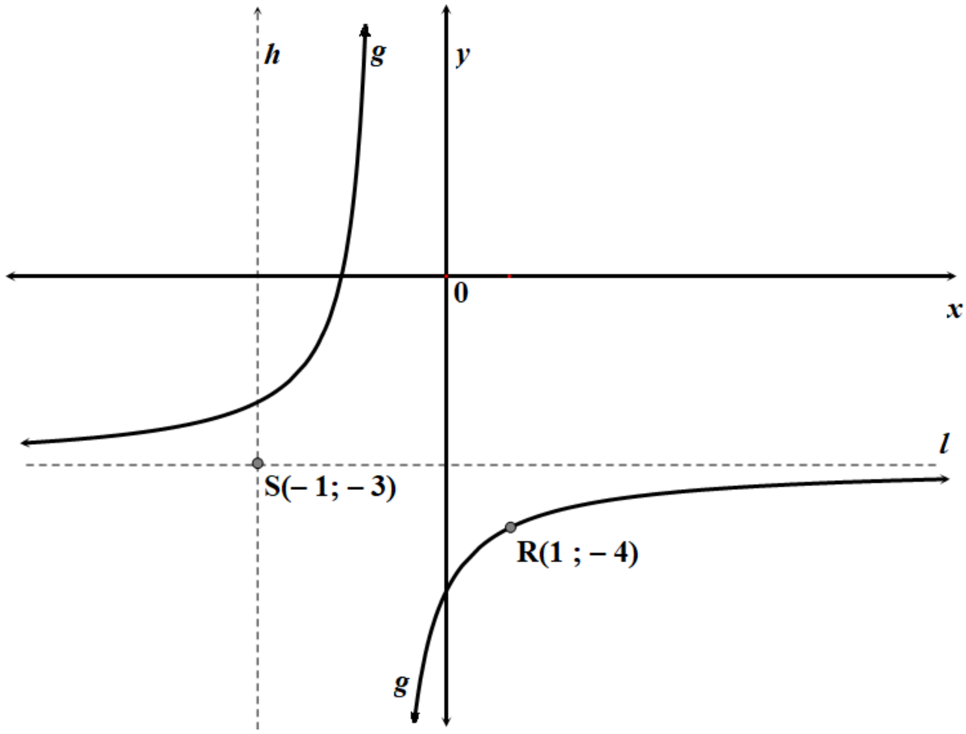
5.5 Prove that $p(x+2) = \frac{1}{4}p(x) - 3$ (3)
[11]

QUESTION 6

The sketch below represents the function defined by: $g(x)=\dfrac{a}{x+p}+q$

Straight lines h and l intersect at the point $S(-1;-3)$.

The point $R(1;-4)$ is on the graph of g.



6.1 Write down the equations of the asymptotes of g. (2)

6.2 Determine the equation of g. (3)

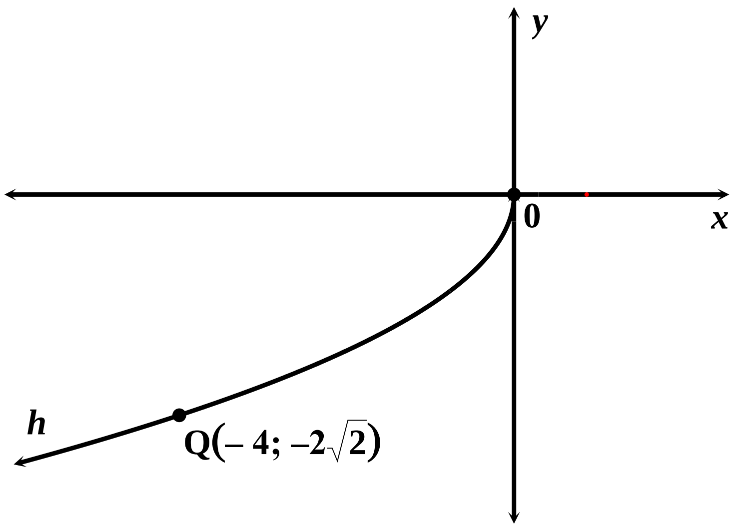
6.3 Determine the equation of a line of symmetry of g with a negative gradient. (2)

6.4 Write down the equations of asymptotes of p if $p(x)=g(x-2)-1$. (2)

[9]

QUESTION 7

Sketched below is the graph h^{-1}, the inverse of a restricted parabola h.
The point $Q\left(-4;-2\sqrt{2}\right)$ lies on the graph of h^{-1}.



7.1 Write down the restricted domain of the parabola h. (1)

7.2 If it is given that, $y=-\sqrt{\frac{x}{a}}$, then show that the equation h^{-1} is $h^{-1}(x)=-\sqrt{-2x}$. (3)

7.3 Write down the equation of h. (2)

7.4 Hence draw the sketch graph of h in your answer book. (2)
[8]

QUESTION 8

8.1 Jasper buys a new car for R245 000. The value of the car depreciates at 13% per annum according to the reducing-balance method. After how many years will the value of the car be R83 543? (3)

8.2 Mr. Williams buys a house for R450 000. He pays a deposit of 10% and takes out a bank loan for the balance.

8.2.1 Calculate the value of the loan. (2)

8.2.2 He pays back the loan by means of equal monthly instalments over a period of 20 years. The first payment is made one month after the allocation of the loan. Interest is calculated at 8% per annum, compounded monthly. Calculate the value of the monthly instalment. (4)

8.2.3 He decides to settle the loan after 17 years. Calculate the outstanding balance on the loan if the last payment is made at the end of the 17th year. (3)
[12]

QUESTION 9

9.1 Determine $f'(x)$ from first principles if it is given that $f(x)=\dfrac{3}{x}$ (5)

9.2 Determine:

9.2.1 $g''(x)$ if $g(x)=\dfrac{1}{2}x^{-2}$ (2)

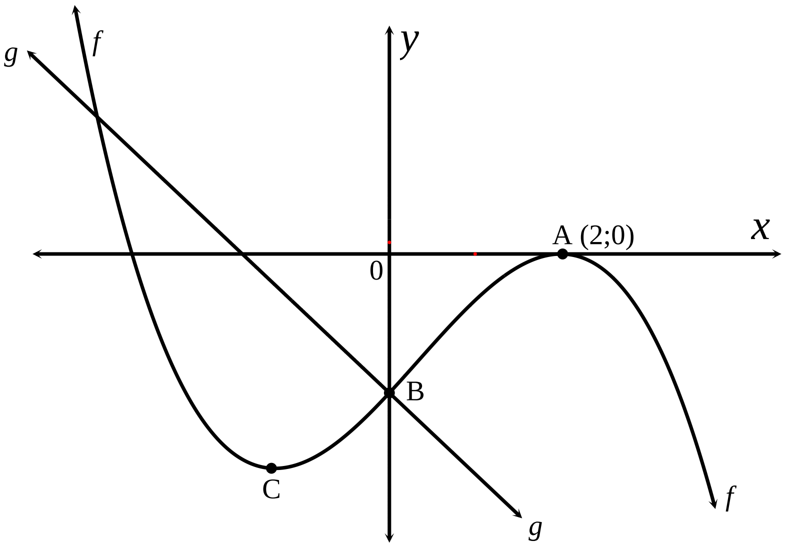
9.2.2 $D_x\left[\sqrt{x}-\dfrac{9-x^2}{x+3}\right]$ (4)

9.2.3 $\dfrac{dx}{dy}$ if $xy=2y^{-3}-\pi y+3$ (4)

[15]

QUESTION 10

The sketch below represents the graphs of the functions defined by:
$f(x)=-x^3+ax^2+bx+c$ and $g(x)=-7x-12$
$A(2;0)$ and C are turning points of f and B is the y-intercept of both f and g.



10.1 Determine the coordinates of B. (2)

10.2 Show that $f(x)=-x^3+x^2+8x-12$ (4)

10.3 Determine the coordinates of C. (4)

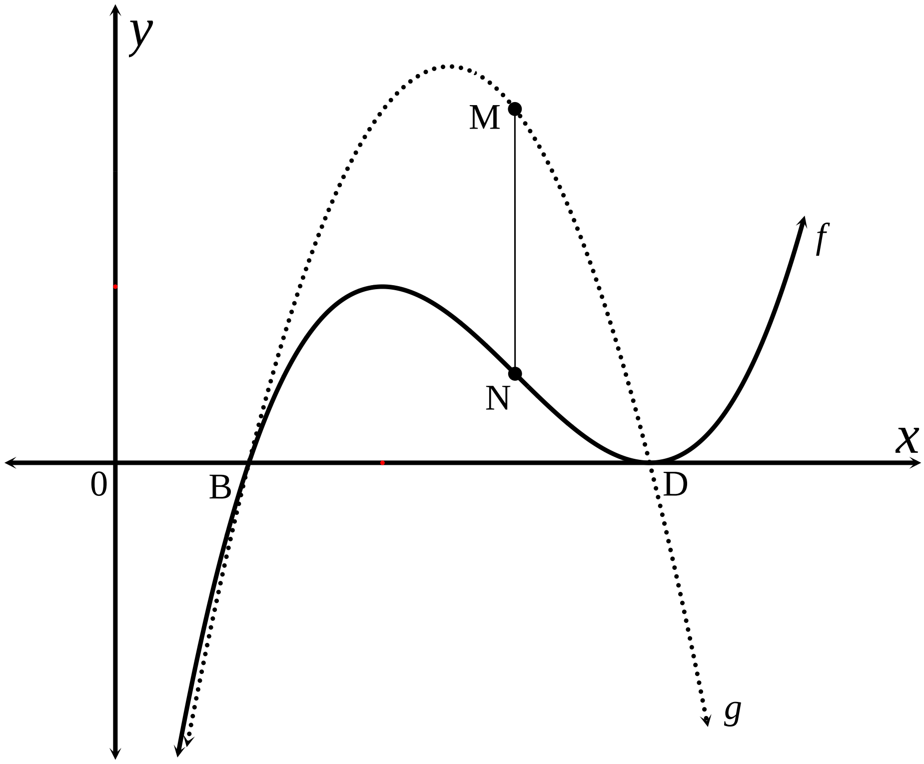
10.4 Write down the values of x for which $f'(x)>0$ (2)

10.5 Is the graph of f concave up or concave down at $(1;-4)$?
Show ALL your calculations. (3)

[15]

QUESTION 11

The sketch below shows the graphs of $f(x)=(x-2)^2(2x-1)=2x^3-9x^2+12x-4$
and $g(x)=ax^2+bx+c$.



11.1 The parabola g, is drawn through B and D (the x-intercepts of f), intersecting the y-axis in $(0;-8)$.

Show that the equation of the parabola is $g(x)=-8x^2+20x-8$. (4)

11.2 A vertical line is now drawn between B and D to intersect the graphs of f and g at M and N respectively.

Show that the length of MN is given by $-2x^3+x^2+8x-4$. (2)

11.3 For which x value(s) between B and D will MN have its maximum length? (4)

[10]

QUESTION 12

12.1 The events A and B are such that $P(A\ or\ B) = 0,75$ and $P(A) = 0,3$.

Determine $P(B)$ if A and B are independent events. (3)

12.2 Samuel's Spaza Shop has 3 strawberry yogurts, 2 peach yogurts, 2 chocolates yogurts and 3 banana yogurts.

12.2.1 In how many different ways can these yogurts be arranged in the display fridge? (2)

12.2.2 What is the probability that all the banana yogurts are next to each other in the display fridge? (3)

12.3 The digits 1 to 7 are used to make 4-digit codes. How many even codes less that 4000 codes can be selected if digits may not be repeated? (3)

[11]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\,\Delta ABC = \frac{1}{2} ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x\,;\,y) \to (x\cos\theta + y\sin\theta\,;\,y\cos\theta - x\sin\theta)$ $(x\,;\,y) \to (x\cos\theta - y\sin\theta\,;\,y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$